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ASYMPTOTIC DISTRIBUTION OF EIGENVALUES FOR SCHRÖDINGER OPERATORS WITH HOMOGENEOUS MAGNETIC FIELDS

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1. Introduction

We consider the Schrödinger operator for one particle subjected to an external potential $-V(x)$, $x=(x_1, x_2, x_3)\in R_x^3$, and a homogeneous magnetic field of magnitude $b>0$ along the x_3 axis. Under a suitable normalization of units, this operator is described in the following form:

$$H=(D_1-bx_2/2)^2+(D_2+bx_1/2)^2+D_3^2-V(x)\,,\quad D_j=-i\partial/\partial x_j\,.$$

If $V(x)\to 0$ as $|x|\to\infty$, then H has essential spectrum beginning at b and discrete spectrum below the bottom b([1]). We denote by $N(\lambda)$, $\lambda>0$, the number of eigenvalues less than $b-\lambda$ of H with repetitoin according to multiplicities. The aim of this paper is to study the asymptotic behavior as $\lambda\to 0$ of $N(\lambda)$ when H has an infinite number of eigenvalues below the bottom b of essential spectrum.

We assume $V(x)$ to satisfy the following

ASSUMPTION $(V)_m$. (i) $V(x)$ is positive and C^1-smooth. (ii) There exists $m>0$ such that

$$C^{-1}\langle x\rangle^{-m}\leqq V(x)\leqq C\langle x\rangle^{-m}\,,\qquad C>1\,,$$

and $|\partial V/\partial x_j|\leqq C_j\langle x\rangle^{-m-1}$, where $\langle x\rangle=(1+|x|^2)^{1/2}$.

We now formulate the main theorem. Under the above assumption, H is essentially self-adjoint in $C_0^\infty(R_x^3)$. We denote by the same notation H its self-adjoint realization in $L^2(R_x^3)$.

Theorem 1. *Assume* $(V)_m$ *and denote by* $N(\lambda)$, $\lambda>0$,, *the number of eigenvalues less than* $b-\lambda$ *of* H.

(i) *If* $0<m<2$, *then*

$$N(\lambda)=N_0(\lambda;\,V)(1+o(1))\,,\qquad \lambda\to 0\,, \tag{1.1}$$