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LOGARITHMIC DEL PEZZO SURFACES OF RANK ONE WITH CONTRACTIBLE BOUNDARIES

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Contents

Introduction. Let k be an algebraically closed field of characteristic zero. Consider a pair (V, D) where V is a nonsingular projective rational surface and D is a reduced effective divisor with only simple normal crossings. We employ the terminology and notations in MT[7 and 8]. By MT[7; Lemma 2.1], there exists a birational morphism $u: (V, D) \to (\tilde{V}, \tilde{D})$ such that $u_* D = \tilde{D}$, $(\tilde{V}, \tilde{D})$ is almost minimal and $\bar{\kappa}(V-D) = \bar{\kappa}(\tilde{V}-\tilde{D})$. In particular, if $\tilde{V}-\tilde{D}$ is affine-ruled, so is $V-D$. The divisor $D+K_V$ can be decomposed into $D+K_V = (D^\# + K_V) + Bk(D)$ (cf. MT[7; §1.5]). Suppose hereafter that (V, D) is almost minimal. Then $\bar{\kappa}(V-D) \geqq 0$ iff $D^\# + K_V$ is numerically effective (cf. MT[7; §1.12]). In this case $D+K_V = (D^\# + K_V) + Bk(D)$ is nothing but the Zariski decomposition.

By Theorem 2.11 in MT[7] and by Main Theorem and Theorem 7 in MT[8], on the case where $\bar{\kappa}(V-D) = -\infty$, $V-D$ is affine-uniruled except the unknown case where (V, D) is a logarithmic del Pezzo surface of rank one with contractible boundaries (cf. Definition 1.1 below). Professor M. Miyanishi conjectured

Conjecture (1) (the weaker form). If (V, D) is a log del Pezzo surface of rank one with contractible boundaries then $V-D$ is affine-uniruled.

Conjecture (2). Let (V, D) be the same as in the conjecture (1). Then there exists a finite subgroup G of $\mathrm{PGL}(2, k) = \mathrm{Aut}_k(\boldsymbol{P}^2)$ such that $\bar{V}$ is isomor-