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GENERALIZATIONS OF NAKAYAMA RING VII

(HEREDITARY RINGS)

Dedicated to Professor Takasi Nagahara on his 60th birthday

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We have studied left serial rings with $(*, 1)$ or $(*, 2)$ in [7] and [8] as a generalization of Nakayama ring (generalized uniserial ring).

In this note, we shall replace the assumption "left serial" to "hereditary", and give, in Sections 2∼5, characterizations of an artinian hereditary ring with $(*, n)$ in terms of the structure of R; $n \leqslant 3$. In Section 6, we shall study another type of hereditary algebras over an algebraically closed field, i.e., right US-n hereditary algebras.

1. Hereditary rings

Throughout this paper we assume that a ring R is a left and right artinian ring with identity. We shall use the notations and terminologies given in [2]∼[8]

First we recall the definition of $(*, n)$.

$(*, n)$ *Every maximal submodule of a direct sum of n hollow modules is also a direct sum of hollow modules* [2] *and* [4]

In this case we may restrict ourselves to a direct sum of hollow modules of a form eR/K, where e is a primitive idempotent and K is a submodule of eR [4].

Let R be an artinian hereditary ring. Then R is isomorphic to the ring of generalized tri-angular matrices over simple rings [1]. We are interested in a hereditary ring with $(*, n)$, and so we may assume that R is basic. Then

$$(1) \qquad R \approx \begin{pmatrix} \Delta_1 & M_{12} \cdots\cdots M_{1n} \\ & \Delta_2 \; M_{23} \cdots M_{2n} \\ & \ddots \qquad \vdots \\ 0 & \ddots \;\; \vdots \\ & \qquad \Delta_n \end{pmatrix}$$

where the Δ_i are division rings and the M_{ij} are left Δ_i and right Δ_j modules. It is clear that $M_{ij} = e_i R e_j$ ($e_i = e_{ii}$ matrix units).