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ON TIME CHANGE OF SYMMETRIC MARKOV PROCESSES

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1. Introduction

Let X be a locally compact separable metric space and m be an everywhere dense positive Radon measure on X. Suppose that we are given an irreducible regular Dirichlet space $(\mathcal{E}, \mathcal{F})$ on $L^2(X; m)$. There then corresponds an m-symmetric Markov process $\boldsymbol{M}=(\Omega, \mathcal{B}, X_t, P_x)$. Let us fix a positiv Radon measure μ charging not a set of zero capacity. Let A be the positive continuous additive functional (PCAF) associated with μ and set $Y_t=X(A^{-1}(t))$. We shall denote by $(\mathcal{E}^\mu, \mathcal{F}^\mu)$ the $L^2(X; \mu)$-Dirichlet space of $\boldsymbol{M}^\mu=(\Omega, \mathcal{B}, Y_t, P_x)$. The purpose of this paper is to characterize the extended Dirichlet space $(\mathcal{E}^\mu, \mathcal{F}^\mu_e)$ of $(\mathcal{E}^\mu, \mathcal{F}^\mu)$. The characterization given here is originally discussed by Silverstein [7], but his proof seems to be insufficient. In the transient case however, Fukushima [3] has established the characterization.

If $(\mathcal{E}, \mathcal{F})$ is transient, then its extended Dirichlet space $(\mathcal{E}, \mathcal{F}_e)$ becomes a Hilbert space continuously embedded in an $L^1(X; \text{gdm})$-space. If $\boldsymbol{M}$ is recurrent in the sense of Harris, then the quotient space of $\mathcal{F}_e$ by the family of constant functions becomes a Hilbert space continuously embedded in an $L^1(X; gdm)$-space. In the latter case, we shall identify $\mathcal{F}_e$ and the quotient space. Let Y be the support of A, γ be the restriction operator to Y and $\mathcal{F}_{X-Y}=\{u\in \mathcal{F}_e; u=0 \text{ q.e. on } Y\}$. Let

$$\mathcal{F}_e = \mathcal{F}_{X-Y}+\mathcal{H}^Y \tag{1.1}$$

be an orthogonal decomposition. Then the main result is, for a suitable choice of the version, $\mathcal{F}^\mu_e=\mathcal{H}^Y$ and $\mathcal{E}^\mu(\gamma u, \gamma u)=\mathcal{E}(u, u)$ for all $u\in\mathcal{H}^Y$.

We shall prove this in section 3 by assuming that $\boldsymbol{M}$ is recurrent in the sense of Harris. See [3] for the same result in transient case. We believe that the present generalization to recurrent case is important because recurrent symmetric Markov processes appear in many applications and besides the present additional condition of the Harris property can be checked by a kind of coerciveness condition on the Dirichlet form ([4]).

In section 4, under the hypothesis that $Y=X$, we shall be concerned with