

STRUCTURAL PROPERTIES OF FUNCTIONAL DIFFERENTIAL EQUATIONS IN BANACH SPACES

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1. Introduction and summary of the results

In the present work we study the structural properties of linear autonomous functional differential equations in Banach spaces within the framework of linear operator theory. We shall explain our motivation of this study.

In a series of papers Bernier and Manitius [4], Manitius [29] and Delfour and Manitius [15] they have developed an excellent state space theory for linear retarded functional differential equations (FDE's) in the product space $\mathbf{R}^n \times L_p([-h, 0]; \mathbf{R}^n)$, $h > 0$. The theory is based on certain relations between semigroups associated with the FDE's and the so-called structural operators F and G . The structural operators have enriched the qualitative theory of linear FDE's and have provided various new and efficient techniques for the study of control