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CERTAIN ASPECTS OF TWISTED LINEAR ACTIONS

Dedicated to Professor Hirosi Toda on his 60th birthday

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0. Introduction

In the previous paper [2], we have introduced the concept of a twisted linear action which is an analytic action of a non-compact Lie group on a sphere, and we have shown as an example that there have been uncountably many topologically distinct analytic actions of $\boldsymbol{SL}(n, \boldsymbol{R})$ on the $(2n-1)$-sphere.

In this paper, we shall show another aspect of twisted linear actions. In particular, we shall show that there are uncountably many C^1-differentiably distinct but topologically equivalent analytic actions of $\boldsymbol{SL}(n, \boldsymbol{R})$ on a k-sphere for each $k \geqq n \geqq 2$.

1. Twisted linear actions

Throughout this paper, a matrix means only the one with real coefficients.

1.1. Let $\boldsymbol{u}=(u_i)$ and $\boldsymbol{v}=(v_i)$ be column vectors in $\boldsymbol{R}^n$. As usual, we define their inner product by $\boldsymbol{u}\cdot\boldsymbol{v}=\sum_i u_i v_i$ and the length of $\boldsymbol{u}$ by $||\boldsymbol{u}||=\sqrt{\boldsymbol{u}\cdot\boldsymbol{u}}$. Let $M=(m_{ij})$ be a square matrix of degree n. We say that M satisfies the condition $(\boldsymbol{T})$ if the quadratic form

$$\boldsymbol{x}\cdot M\boldsymbol{x} = \sum_{i,j} m_{ij} x_i x_j$$

is positive definite. It is easy to see that M satisfies $(\boldsymbol{T})$ if and only if

$$(\boldsymbol{T}') \quad \frac{d}{dt}||\exp(tM)\boldsymbol{x}||>0 \quad \text{for each} \quad \boldsymbol{x}\in\boldsymbol{R}_0^n=\boldsymbol{R}^n-\{0\},\ t\in\boldsymbol{R}\,.$$

If M satisfies $(\boldsymbol{T}')$, then

$$\lim_{t\to+\infty}||\exp(tM)\boldsymbol{x}||=+\infty \quad \text{and} \quad \lim_{t\to-\infty}||\exp(tM)\boldsymbol{x}||=0$$

for each $\boldsymbol{x}\in\boldsymbol{R}_0^n$, and hence there exists a unique real valued analytic function τ

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