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SUPER DIFFERENTIAL CALCULUS

KATSUMI YAGI

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The theory of super differential manifolds has been developed in recent years by many authors. While there are several approaches to the subject, we shall take the so-called geometric approach, developed by Boyer-Gitler [2], DeWitt [3] and Rogers [4]. On the other hand, Bernshtein-Rosenfel'd [1] have introduced a differential calculus on an infinite dimensional Euclidean space. Regarding the super Euclidean space as a projective limit of a family of finite dimensional Euclidean spaces as [1], in this note we shall propose a super differential calculus as a foundation for the theory of super manifolds. The underlying principle is to describe the concept of super differential calculus in terms of a differential calculus on an infinite dimensional Euclidean space. This gives a reduction of a "super" argument to an ordinary one and leads to an easier and clearer understanding of the theory of super manifolds. In section 4 we obtain the Cauchy-Riemann equations for a super smooth function, first obtained in [2], which is rather simplified. In the last section the inverse mapping theorem is also obtained following our principle.

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1. Super numbers and super Euclidean spaces

Let $\{\zeta^N : N \geqq 1\}$ be a set of countably infinite distinct letters. These are fixed once for all. We denote by Λ_N the Grassmann algebra of the vector space generated by $\{\zeta^1, \zeta^2, \cdots, \zeta^N\}$ over the real number field $\boldsymbol{R}$ where for $N=0$, Λ_0 denotes the real number field $\boldsymbol{R}$. The family $\{\Lambda_N : N \geqq 0\}$ and the natural projection of Λ_N onto Λ_{N-1} define the projective limit, denoted by Λ. In a natural way, Λ can be identified with the algebra of all formal series of the following form:

$$z = \sum_{K \in \Gamma} z_K \zeta^K$$

where Γ denotes the set of all h-tuples $K=(k_1, k_2, \cdots, k_h)$ of integers ($h \geqq 0$) with $1 \leqq k_1 < k_2 < \cdots < k_h$ and $z_k \in \boldsymbol{R}$ and $\zeta^K = \zeta^{k_1} \zeta^{k_2} \cdots \zeta^{k_h}$ ($\zeta^\phi = 1 \in \boldsymbol{R}$). The algebra