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ON THE VOLUME OF POSITIVELY CURVED KÄHLER MANIFOLDS

Dedicated to Professor Shingo Murakami on his 60th birthday

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1. Introduction

In Riemannian geometry, it is a fundamental question to ask how the geometric invariants of Riemannian manifolds are influenced by curvature restrictions.

The volume of Riemannian manifolds and of geodesic balls in them is one of the most basic invariants, for which Bishop-Gromov's comparison theorem is well known (see [3], [5]).

In this note, we prove Bishop-Gromov type comparison theorem for the class of complete connected Kähler manifolds of complex dimension n whose Ricci curvature and holomorphic sectional curvature satisfy

$$\mathrm{Ric} \geqq 2(n+1)\,\delta^2$$

and

$$K_H \geqq 4\,\delta^2\,,$$

respecitvely, where δ is a positive real number (Theorem 1). Moreover, we characterize the complex projective space with the canonical metric by its volume among this class (Theorem 2).

In Section 2 we prepare notations and some preliminary results. Here we reduce our problem to the estimation of the mean curvature of geodesic spheres, which is carried out in Section 3. In Section 4 we estimate the volume element with respect to polar coordinates and prove our main theorems.

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2. Notations and preliminaries

Let M be a complete connected Riemannian manifold of dimension $m(m\geqq$