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COBORDISM CLASSIFICATION OF KNOTTED HOMOLOGY 3-SPHERES IN S^5

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In [8], odd dimensional knot cobordism groups C_{2n-1} ($n \geqq 2$) were studied and given a purely algebraic description. C_{2n-1} is the set of all cobordism classes of knotted $(2n-1)$-spheres in the $(2n+1)$-sphere, and it is an abelian group under connected sum. In this paper we extend this to knotted homology 3-spheres in the 5-sphere and consider their homology cobordism classes. The set of all such homology cobordism classes, denoted by C_3^H, is also an abelian group under connected sum. We shall describe the group C_3^H using the usual homology cobordism group of homology 3-spheres and Levine's cobordism group of certain matrices (Theorem 1.1). Our argument heavily depends on that of J. Levine.

In §3 we introduce the bounding genus and the Murasugi number of a knotted homology 3-sphere in S^5. Using our techniques we can estimate these numerical invariants from above. In §4 we consider the case of algebraic 3-knots which appear around isolated singularities of complex hypersurfaces in $\boldsymbol{C}^3$ (see [11]). For example, we shall show that two algebraic 3-knots which are homology 3-spheres and defined by weighted homogeneous polynomials are homology cobordant if and only if they are isotopic.

Throughout the paper, we shall work in the C^∞ category. Homology groups will always be with integral coefficients.

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1. Statement of the main result

A *3-knot* will denote the oriented isotopy class of an embedded homology 3-sphere in the 5-sphere S^5. Two 3-knots K_0, K_1 are *homology cobordant* if there exists a compact oriented 4-submanifold W of $[0, 1] \times S^5$ with $\partial W = (1 \times K_1) \cup (0 \times (-K_0))$ such that the inclusion induces an isomorphism $H_*(j \times K_j) \to H_*(W)$ for $j=0, 1$. We call W a *homology cobordism* between K_0 and K_1. If W is diffeomorphic to $[0, 1] \times K_0$, we say that K_0 is *concordant* to K_1. A 3-knot