Ohmori, Z.
Osaka J. Math.
25 (1988), 149–159

ON THE SCHUR INDICES OF CERTAIN IRREDUCIBLE CHARACTERS OF REDUCTIVE GROURS OVER FINITE FIELDS

ZYOZYU OHMORI

(Received September 11, 1986)

Introduction. Let $\boldsymbol{F}_q$ be a finite field with q elements, of characteristic p. Let G be a connected, reductive linear algebraic group defined over $\boldsymbol{F}_q$, with Frobenius endomorphism F, and let G^F denote the group of F-fixed points of G. In [13], we investigated, under the assumption that the centre Z of G is connected, the rationality-properties of the characters λ^{G^F} of G^F induced by certain linear characters λ of a Sylow p-subgroup of G^F and, using the results obtained there, proved some propositions concerning the Schur indices of the semisimple or regular irreducible characters of G^F. In this paper, we shall treat the general case, that is, the case that Z is not necessarily connected. The main results are stated and proved in § 2. In particular, we get the following (see Corollary 1 to Proposition 1, § 2):

Theorem. *Any irreducible Deligne-Lusztig character* $\pm R_T^\theta$ *of* G^F ([4]) *has the Schur index at most two over the field* $\boldsymbol{Q}$ *of rational numbers.*

I wish to thank Profesosr N. Iwahori who kindly taught me properties of the Cartan matrices. I also thank Professor S. Endo for his kind advices during the preparation of the paper. The referee gave me valuable comments for the old version of the paper. Finally, I wish to dedicate this paper to the late Professor T. Miyata.

1. Some lemmas. Let G and F be as above. Let B be an F-stable Borel subgroup of G with the unipotent radical U and T an F-stable maximal torus of B. For a root α of G (with respect to T), let U_α denote the root subgroup of G associated with α. Let $U.$ be the subgroup of U generated by the non-simple positive root subgroups U_α (the ordering on the roots is the one determined by B). Then $U/U.$ is commutative and can be regarded as the direct product $\prod_{\alpha\in\Delta} U_\alpha$, where Δ is the set of simple roots. As $FU.=U.$, F acts on $U/U.=\prod_{\alpha\in\Delta} U_\alpha$ and this action is the one induced by the maps $F\colon U_\alpha\to FU_\alpha$, $\alpha\in\Delta$. Let ρ be the permutation on the roots α given by $FU_\alpha=U_{\rho\alpha}$ and let I