Gow, R.
Osaka J. Math.
25 (1988), 135-147

REAL 2-BLOCKS OF CHARACTERS OF FINITE GROUPS

RODERICK GOW

(Received September 5, 1986)

Let G be a finite group and let p be a prime divisor of $|G|$. Let B be a p-block of irreducible complex characters of G. It is straight forward to show that the complex conjugates of the irreducible characters in B form another p-block of G, $\bar{B}$, say. B is said to be a real p-block if $B=\bar{B}$. Brauer's concept of the defect groups of a block is well known and has been extensively investigated. In this paper, we will show that if B is a real 2-block of G, there is a related concept of extended defect groups of B, which can be explained in the following manner. It is easily proved that a non-principal real 2-block has at least one non-identity real defect class, K, say. The Sylow 2-subgroups of the extended centralizers of the elements of K comprise a complete conjugacy class of 2-groups, which we call the extended defect groups of K. We then show that for a non-principal real 2-block, we obtain the same conjugacy class of 2-groups for any choice of real defect class and associated extended defect groups. This unique conjugacy class will be referred to as the class of extended defect groups of the real 2-block. The principal 2-block is also real but is somewhat anomalous in that it is the only real 2-block of maximal defect and its unique real defect class is the identity class.

We obtain various general results on the number of real 2-blocks and, in particular, show that if 2^a is the 2-part of $|G|$, the number of real blocks of defect $a-1$ equals the number of real 2-regular classes of defect $a-1$. We also show that, if B is a real 2-block of defect zero and w is an involution that generates an extended defect group of B, then there exists a Sylow 2-subgroup Q of G with $Q\cap Q^w=1$. This generalizes part of a well-known theorem of Green on defect groups and Sylow intersections.

We mention here one example of how the conjugacy theorem for extended defect groups yields group-theoretic information. Suppose that G is a simple group of Lie type in characteristic 2. The block of G corresponding to the Steinberg character of G is a real block of defect zero. Let w be an involution that generates an extended defect group of the block. Using properties of the Steinberg character, we can prove that any real element of odd order in G must be inverted by a conjugate of w, and consequently can be written as a product of two involutions conjugate to w. Our Sylow intersection theorem for real 2-