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LEFT SERIAL RINGS OVER WHICH EVERY RIGHT MODULE WITH HOMOGENEOUS TOP IS A DIRECT SUM OF HOLLOW MODULES

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Let R be a left and right artinian ring with identity, and J the Jacobson radical of R. In [4], M. Harada has considered a left serial ring R satisfying a condition $(*, 2)$ that every maximal submodule of a direct sum of any two hollow modules is also a direct sum of hollow modules, and characterized such a ring by the structure of eR for each primitive idempotent e. Further it has been shown that the condition $(*, 2)$ is equivalent to saying that every factor module of $eJ\oplus eR$ is a direct sum of hollow modules for every primitive idempotent e. Modifying this, we here consider the following condition on a projective indecomposable right module eR over a ring R.

(A): *Every factor module of* $eR\oplus eR$ *is a direct sum of hollow modules.*

Clearly if R is a ring of right local type, then all projective indecomposable right R-modules satisfy the condition (A), and as well known ([6]), R is left serial. The purpose of this paper is to characterize left serial rings over which every projective indecomposable right module eR satisfies the condition (A) (i.e. rings R in the title (see Theorem 1 for the equivalence)) in terms of the structure of eR. Thus our result gives a generalization of rings of right local type.

In the first section we consider various conditions equivalent to (A) (Theorem 1). In particular, the condition 4) of Theorem 1 which is described in terms of homomorphisms between factor modules of eR is frequently used later to check whether eR satisfies (A) or not. We assume in the second and third section that R is a left serial ring. In the second section we shall give some properties induced from the condition (A) to prepare the proof of the main theorem. In the third section we give the main theorem (Theorem 2). In the last section we give some examples.

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