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ON A RELATION BETWEEN HIGHER ORDER ASYMPTOTIC RISK SUFFICIENCY AND HIGHER ORDER ASYMPTOTIC SUFFICIENCY IN A LOCAL SENSE

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1. Introduction. In Takeuchi [4] higher order asymptotic risk sufficiency of maximum likelihood estimator has been discussed. In this paper we try to find some relations between asymptotic risk sufficiency with a special loss function and asymptotic sufficiency in a local sense.

Let $\mathcal{P}_n=\{P_{\theta,n};\theta\in\Theta\}$ be a family of probability distributions on a measurable space $(\mathcal{X}, \mathcal{A}_n)$ with an index set Θ which is a subset of an Euclidean space with the usual norm $|\cdot|$. For a sub σ-field $\mathcal{C}$ of $\mathcal{A}_n$, real number $c\geqq 0$ and $\theta, \theta'\in\Theta$ let $r_n^{\mathcal{C}}(c: \theta, \theta')=\inf\ (1+c)^{-1}\{1-E_{P_{\theta,n}}(\phi)+cE_{P_{\theta',n}}(\phi);\ \phi$ are $\mathcal{C}$-measurable statistical test functions on $\mathcal{X}\}$. We note that $r_n^{\mathcal{C}}(c: \theta, \theta')$ means the Bayes risk of statistical problem of testing a hypothesis '$P_{\theta',n}$ is true' against an alternative '$P_{\theta,n}$ is true' with experiment $(\mathcal{X}, \mathcal{C}, \{P_{\theta',n}, P_{\theta,n}\})$ relative to a prior probability distribution $(c/(1+c), 1/(1+c))$ on $\{\theta', \theta\}$ provided that the loss function is simple.

Let $\{\mathcal{B}_n; n=1, 2, \cdots\}$ be a sequence of sub σ-fields of $\{\mathcal{A}_n\}$ $(\mathcal{B}_n\subset\mathcal{A}_n)$. In this paper we give a sufficient condition about the Bayes risk $r_n^{\mathcal{B}_n}$ for $\{\mathcal{B}_n\}$ to be higher order locally asymptotically sufficient sequence of σ-fields. More precisely our main result in this paper is the following: Under some conditions if for some positive number α $\sup_{c>0}\sup_{\theta^*\in K}\sup_{\theta\,:\,n^{1/2}|\theta-\theta^*|\leqq b}\{r_n^{\mathcal{B}_n}(c: \theta, \theta^*)-r_n^{\mathcal{A}_n}(c: \theta, \theta^*)\}=o(n^{-\alpha})$ for every $b>0$ and every compact subset K of Θ, then for every β satisfying $0<\beta<3^{-1}\alpha$ $\{\mathcal{B}_n\}$ is locally asymptotically sufficient for $\{\mathcal{P}_n\}$ with order $o(n^{-\beta})$ in the sense that for each $n=1, 2, \cdots$ and each $\theta_0\in\Theta$ there exists a family $\{Q_{\theta,n}^{\theta_0}; \theta\in\Theta\}$ of probability distributions on $(\mathcal{X}, \mathcal{A}_n)$ for which $\mathcal{P}_n$ is sufficient σ-field and that for every $b>0$

$$\sup_{\theta\,:\,n^{1/2}|\theta-\theta_0|\leqq b}||P_{\theta,n}-Q_{\theta,n}^{\theta_0}||_{\mathcal{A}_n}=o(n^{-\beta})$$

uniformly in θ_0 over every compact subsets of Θ. Here $||\cdot||_{\mathcal{A}_n}$ means the total variation norm over $\mathcal{A}_n$.

We have discussed such a problem in the case $\alpha=\beta=0$ in Suzuki [3] under