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FORMAL POWER SERIES SOLUTIONS OF THE STATIONARY AXISYMMETRIC VACUUM EINSTEIN EQUATIONS

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Introduction

The present paper is concerned with the formal power series solution of the stationary axially symmetric vacuum (SAV) Einstein equations. By using a formal Riemann-Hilbert problem we shall present an explicit formula representing the formal solution in terms of a certain $\infty\times\infty$ matrix ξ. The space V_0 of all formal solutions corresponding to the SAV space-times which has the formal Ernst potentials will be determined exactly. The asymptotically flat SAV space-times constitute a subset of V_0.

A four-dimensional manifold M with a Lorentz metric g is said to be stationary axially symmetric if the metric g is time-independent and invariant under the rotation around an axis. Let R_{ij} be the Ricci tensor of the metric g. In addition to being stationary axially symmetric, if the Einstein equations $R_{ij}=0$ hold on M, then the space (M, g) is called a SAV space-itme. In case (M, g) is a SAV space-time, by an appropriate choice of the local coordinates (z, ρ, x^1, x^2) the Einstein equations are reduced to the differential equation for a 2×2 matrix function h of the variables, z and ρ (cf. Belinskii-Zakharov [1])

$$(1)\qquad \partial_\rho(\rho\partial_\rho h\cdot h^{-1})+\partial_z(\rho\partial_z h\cdot h^{-1})=0$$

where $\partial_z=\partial/\partial_z$ and $\partial_\rho=\partial/\partial_\rho$. Further the geometrical restrictions are imposed on h: the matrix h is symmetric and $\det(h)=-\rho^2$. We shall consider the formal power series solution $h\in\mathfrak{gl}(2, \boldsymbol{R}[[z, \rho]])$ whose twist potentials are also elements of $\mathfrak{gl}(2, \boldsymbol{R}[[z, \rho]])$ (cf. Section 1). The asymptotically flat SAV space-times are included in this category (cf. Introduction of Hauser-Ernst [6]).

Recently, much progress has been made on the inverse scattering approach to equation (1), see Hauser-Ernst [5], [6], Belinskii-Zakharov [1] and Maison [7]. See also Cosgrove [2] for the relationship between these works. In these

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