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Osaka J. Math.
25 (1988), 1-18

EXISTENCE AND SMOOTHNESS FOR CERTAIN DEGENERATE PARABOLIC BOUNDARY VALUE PROBLEMS

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(Received July 21, 1986)

1. In [6], S. Ito has considered the following parabolic initial-boundary value problem:

$$\frac{\partial u}{\partial t}=\sum_{i,j=1}^{n}a_{ij}(x,t)\frac{\partial^2 u}{\partial x_i\partial x_j}+\sum_{i=1}^{n}b_i(x,t)\frac{\partial u}{\partial x_i}+c(x,t)u \tag{1.1}$$

in $\Omega\times R_+$ with $\Omega\subset R^n$,

$$\alpha(x,t)\frac{\partial u}{\partial n}(x,t)+\beta(x,t)u(x,t)=f(x,t)\qquad\text{on}\quad\partial\Omega\times R_+ \tag{1.2}$$

where $\partial/\partial n$ is the derivation in the direction of outer co-normal and

$$\alpha(x,t)\geq 0,\ \beta(x,t)\geq 0,\ \alpha(x,t)+\beta(x,t)=1\,, \tag{1.3}$$

and

$$u(x,0)=u_0(x)\qquad\text{for}\quad x\in\Omega\,. \tag{1.4}$$

He solved the problem by constructing explicitly a fundamental solution. We wish to apply instead the well-known method of reducing boundary value problems to pseudo-differential problems on the boundary [5, Chapter XX for the elliptic case, and [3,10] for the parabolic case]. In [7] we have analyzed in this manner the corresponding degenerate elliptic boundary value problem and established the hypoellipticity of the appropriate pseudo-differential boundary operator; in fact, under assumptions such as (1.3), one easily sees that this operator satisfies the condition for the existence of a parametrix with symbol in a suitable space $S^m_{\rho,\delta}(\partial\Omega)$. The parabolic case, to be considered in the sequel, differs from the elliptic case in two respects: (i) The boundary operator induced by (1.2) is no longer invertible in an $S^m_{\rho,\delta}(\partial\Omega\times R_+)$ space - one needs weighted classes of symbols as introduced in [1] (in fact vector weights are required; (ii) The manifold $\partial\Omega\times R_+$ is a manifold *with boundary* $\partial\Omega\times\{0\}$ and a Cauchy problem for a pseudo-differential equation has to be solved. We deal with