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REGULARITY IN TIME OF THE SOLUTION OF PARABOLIC INITIAL-BOUNDARY VALUE PROBLEM IN L^1 SPACE

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1. Introduction

This paper is concerned with the regularity in t of the solution of the initial-boundary value problem of the linear parabolic partial differential equation

$$(1.1)\qquad \partial u(x, t)/\partial t+A(x, t, D)u(x, t) = f(x, t)\,, \qquad \Omega\times(0, T]\,,$$

$$(1.2)\qquad B_j(x, t, D)u(x, t) = 0\,,\quad j = 1, \cdots, m/2\,, \qquad \partial\Omega\times(0, T]\,,$$

$$(1.3)\qquad u(x, 0) = u_0(x)\,, \qquad \Omega\,.$$

Here Ω is a not necessarily bounded domain in R^N with boundary $\partial\Omega$ satisfying a certain smoothness hypothesis. For each $t\in[0, T]$ $A(x, t, D)$ is a strongly elliptic linear differential operator of order m, and $\{B_j(x, t, D)\}_{j=1}^{m/2}$ is a normal set of linear differential operators of respective orders $m_j<m$. It is assumed that the realization $-A_p(t)$ of $-A(x, t, D)$ in $L^p(\Omega)$ under the boundary conditions $B_j(x, t, D)u|_{\partial\Omega}=0, j=1, \cdots, m/2$, generates an analytic semigroup in $L^p(\Omega)$ for any $p\in(1, \infty)$. A sufficient condition for that, which is also necessary when $p=2$, is given in S. Agmon [1]. Assuming moreover that the coefficients of $A(x, t, D)$, $\{B_j(x, t, D)\}_{j=1}^{m/2}$ and some of their derivatives in x belong to Gevrey's class $\{M_k\}$ ([4], [6], [7]) as functions of t and f also belongs to the same class as a function with values in $L^1(\Omega)$, we show that the same is true of the solution of (1.1)–(1.3) considered as an evolution equation in $L^1(\Omega)$ for any initial value $u_0\in L^1(\Omega)$. It should be noted here that if $m_j=m-1$, the boundary condition $B_j(x, t, D)u|_{\partial\Omega}=0$ is satisfied only in a variational sense.

In order to prove the result stated above we show that there exist positive constants K_0, K such that

$$(1.4)\qquad \|(\partial/\partial t)^n(A(t)-\lambda)^{-1}\|\leqq K_0K^nM_n/|\lambda|$$

for any $n=0, 1, 2, \cdots, t\in[0, T]$ and λ in the sector Σ: $|\arg\lambda|\geqq\theta_0$, $0<\theta_0<\pi/2$, where $A(t)$ is the realization of the operator $A(x, t, D)$ in $L^1(\Omega)$ under the boundary conditions $B_j(x, t, D)u|_{\partial\Omega}=0, j=1, \cdots, m/2$. Once (1.4) is established, one