Morita, T.
Osaka J. Math.
24 (1987), 887–909

# A FLUCTUATION THEOREM ASSOCIATED WITH CAUCHY PROBLEMS FOR STATIONARY RANDOM OPERATORS

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(Received June 11, 1986)

## 1. Introduction

Let $\{H^p; p\in \boldsymbol{R}\}$ be a family of separable real Hilbert spaces which are modeled on the Sobolev spaces on a compact manifold without boundary. Consider a stationary process $L(\omega, t)$ on a probability space $(\Omega, \mathcal{F}, P)$ with values in a certain class of linear operators on $H^{-\infty}=\bigcup_p H^p$, which are modeled on pseudo-differential operators. Denote by $L$ the mean operator of $L(\omega, t)$. We assume that the following abstract Cauchy problems are 'well-posed':

$$(1.1)\qquad \begin{cases} \dfrac{du(t)}{dt} = L\Big(\omega, \dfrac{t}{\varepsilon}\Big)u(t) \\ u(0) = u_0\in H^p\,, \end{cases}$$

and

$$(1.2)\qquad \begin{cases} \dfrac{du(t)}{dt} = Lu(t) \\ u(0) = u_0\in H^p\,. \end{cases}$$

The aim of this paper is to investigate the fluctuation of $u^\varepsilon(\omega, t)$ around $u^0(t)$ where $u^\varepsilon(\omega, t)$ and $u^0(t)$ are the solutions of (1.1) and (1.2) respectively. Precisely, let $C([0, T]\to H^q)$ be the space of all continuous functions on $[0, T]$ with values in $H^q$, for $q\in\boldsymbol{R}$. Under the assumption (A.I), (A.II), and (A.III) in Section 2, we show that for any $T>0$, the stochastic process $X^\varepsilon(\omega, t)=\dfrac{u^\varepsilon(\omega, t)-u^0(t)}{\sqrt{\varepsilon}}$ converges weakly to a Gaussian process $X^0(\omega, t)$ in the sense of distribution on $C([0, T]\to H^q)$ for any $q\leq p-\alpha$, where $\alpha$ is determined by the assumptions.

A mathematical motivation of this paper was taken from Khas'minskii's work [8]. We summarize his work here. Let $F(\omega, t, x)$ be a strongly mixing process which is a twice differentiable vecter field on $\boldsymbol{R}^d$ for each $\omega$ and $t$. Let $F(x)$ be the vector field defined as the mean of the process $F(\omega, t, x)$ in some sense. He considered the following Cauchy problems