Ichinose W.
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THE CAUCHY PROBLEM FOR SCHRÖDINGER TYPE EQUATIONS WITH VARIABLE COEFFICIENTS

WATARU ICHINOSE*

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0. Introduction. In this paper we study the Cauchy problem for Schrödinger type equations with variable coefficients

$$(0.1)\qquad \begin{cases} Lu(t,x)\equiv \dfrac{1}{i}\,\partial_t u(t,x)-\dfrac{1}{2}\sum_{j,k=1}^{n}\partial_{x_j}(g^{jk}(x)\,\partial_{x_k}u)+ \\ \qquad\qquad \sum_{j=1}^{n} b^j(x)\,\partial_{x_j}u+c(x)\,u=f(t,x)\quad (x\in R^n)\,, \\ u(0,x)=u_0(x)\,, \end{cases}$$

where $g^{jk}(x)$, $b^j(x)$ and $c(x)$ are in $\mathcal{B}^\infty(R^n)$. We suppose that

(0.2) $g^{jk}(x)$ $(j, k=1, 2, \cdots, n)$ are real valued and satisfy $g^{jk}(x)=g^{kj}(x)$

and that the uniform ellipticity

$$(0.3)\qquad \delta^{-1}|p|^2\leq |\sum_{j,k=1}^{n} g^{jk}(x)\,p_j\,p_k|\leq \delta|p|^2$$

with a positive constant δ. First of all, remark that it is impossible to consider the well posedness of (0.1) in $C^\infty(R^n)$ space, because (0.1) has an infinite propagation speed (see [10]). Therefore, in the present paper we shall consider the well posedness of (0.1) in the sense of L^2. We denote the set of all L^2 valued continuous functions in $t\in[0, T]$ by $\mathcal{E}_t^0([0, T]; L^2)$. We adopt the following definition.

DEFINITION 0.1. We say that *the Cauchy problem* (0.1) *is* L^2 *well posed on* $[0, T_0]$ $(T_0>0)$ (*resp.* $[T_0, 0]$ $(T_0<0)$), if the following is valid for each $T\in(0, T_0]$ (resp. $[T_0, 0)$). For any $u_0(x)\in L^2$ and any $f(t, x)\in\mathcal{E}_t^0([0, T]; L^2)$ (resp. $\mathcal{E}_t^0([T, 0]; L^2)$) there exists one and only one solution $u(t, x)$ of (0.1) in $\mathcal{E}_t^0([0, T]; L^2)$ (resp. $\mathcal{E}_t^0([T, 0]; L^2)$).

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