

GENERALIZATIONS OF NAKAYAMA RING VI

(RIHGT US- n RINGS; $n=3, 4$)

Dedicated to Professor Hiroyuki Tachikawa on his 60th birthday

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We have studied artinian right US-3 rings in [5] and right US-4 algebras over an algebraically closed field in [7]. We shall continue, in this paper, to study a right US-3 (resp. US-4) ring R when R is either hereditary or left serial.

In the first two sections, we shall give the characterization of a right US-3 (resp. US-4) ring R , when R satisfies a weaker condition $(*, 1')$ (see § 1) than R being either hereditary or left serial. In the next two sections, we shall specify the characterizations given in the previous sections to hereditary rings and left serial rings. We shall exhibit several examples in the final section to illustrate the above characterizations.

1. US-3 rings

Throughout this paper we deal with an artinian ring R and every R -module is a unitary right R -module. We shall use the same terminologies and definitions given in [2]~[8].

As a generalization of right serial rings, we considered

$(**, n)$ *Every maximal submodule in a direct sum D of n hollow modules contains a non-zero direct summand of D [5].*

It is clear that if $D/J(D)$ is not homogeneous, D satisfies $(**, n)$. Hence we may restrict ourselves to hollow modules of a form eR/E , where e is a primitive idempotent and E is a submodule of eR . If $(**, n)$ holds for any direct sum of n hollow modules, we call R a *right US- n ring* [5]. Since the concept of US- n ring is Morita equivalent, we study always a basic ring.

We studied right US- n algebras over an algebraically closed field for $n=3$ and 4 in [5] and [7], respectively. In this and next sections we shall give a complete list of the structure of right US-3 (resp. US-4) rings with $(*, 1')$ below. We can give theoretically the complete structure, however as we know a few properties of division rings, we can not give the complete examples for each structure.