Marumoto Y.
Osaka J. Math.
24 (1987), 759-783

SOME HIGHER DIMENSIONAL KNOTS

Dedicated to Professor Itiro Tamura on his 60 th birthday

YOSHIHIKO MARUMOTO

(Received June 13, 1986)

Introduction

In this paper, we construct some classes of higher dimensional knots, and investigate geometrical and algebraic properties of the knots.

For classical knots, many concrete examples are known and studied. On the other hand, for higher dimensional cases, not so many examples, or constructions, are known. One of reasons for this difference is seemed to be derived from the existence of unknotting operations for classical knots to change into the unknot, that is, any 1-knot is changed to the unknot by exchanging the crossings suitably. In [5], F. Hosokawa and A. Kawauchi study an unknotting operation for 2-knots, and recently Kawauchi argues this from more general points of view in [11]. The author does not know whether there exist simple unknotting operations for any n-knots.

In § 1, we first give modifications which change some knots to the unknot, and we call such knots to be of type p, then we prove that *any 2-knot is of type* 2 in Theorem 1.2. Thus our defining 'unknotting operation' is valid for any 2-knots. We have a relationship between n-knots of type p and some disk pairs in Theorem 1.10, and this is very useful in the later sections.

In § 2, we show that *an n-knot of type p is also of type* $(n-p+1)$, and this is a geometrical description of the algebraic duality.

In § 3, we first generalize the notions of semi-unknotted manifolds and ribbon maps [24]. Then we argue relationships between bounding manifolds of knots and immersed disks.

In § 4, we discuss knots of type p and the bounding manifolds. Combining results in §§ 3 and 4, we can conclude that *any 2-knot is the boundary of an immersed disk with only double points singularities* (Corollary 4.2.2).

In §§ 5 and 6, we calculate the Alexander modules of knots, and introduce some algebras to obtain an exact sequence containing a semi-group of knots.

0. Preliminaries

Throughout the paper, we shall work in the piecewise linear category, and