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Osaka J. Math.
24 (1987), 745-758

THE KAUFFMAN POLYNOMIAL OF LINKS AND REPRESENTATION THEORY*

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(Received July 3, 1986)

1. Introduction. In 1984, V. Jones [8] introduced a new polynomial invariant of link isotopy types which is now called the (one variable) Jones polynomial, which was subsequently generalized to the two variable Jones polynomial [4], [13]. These invariants are closely related to the traces of irreducible representations of Iwahori algebras (or Hecke algebras) [2], [7] associated with the symmetric groups (see [5], [9]).

The purpose of this paper is to show that the Kauffman polynomial [12] can also be interpreted as a function F on a certain associative algebra. We define a *knit semi-group* D_n of degree n which is generated by the generators of the braid group B_n on n strings and elements $e_1, e_2, \cdots, e_{n-1}$ in Figure 4. We call an element of D_n an *n-knit*. We get a link $d^\wedge$ in the 3-sphere by closing an n-knit d. We call $d^\wedge$ a *closed n-knit* coming from d. We also define an algebra $E_n(\alpha, \beta)$ for non-zero complex numbers $\alpha, \beta \in \boldsymbol{C}-\{0\}$ as a quotient of a semi-group algebra $\boldsymbol{C}[D_n]$ of D_n over $\boldsymbol{C}$. Then the Kauffman polynomial of a closed n-knit is obtained through $E_n(\alpha, \beta)$.

From Section 7 on, we treat the case $n=3$. Then we can show that the function F is a sum of traces of irreducible representations of the algebra $E_3(\alpha, \beta)$ (Theorem 10.1). The author expects the same is true for general n. In Sections 12–16, we apply our formula to closed 3-braids and 2-bridge links, since they are special types of closed 3-knits. For example, if two closed 3-braids have the same writhe (or twist number) and the same Jones polynomial, they also have the same Q-polynomial (Theorem 13.1). Thus for the closed 3-braid, the Alexnader polynomial and the writhe determine the two variable Jones, Jones, and Q-polynomials. In the actual claculation of the examples in Sections 14 b), c) and 16, the author used a personal computer (NEC PC-9801) with muMATH-83 (Symbolic Mathematics Package) for MS-DOS.

Acknowledgments. I am profoundly indebted to A. Gyoja who gave me many informations about Iwahori algebras. As well, the notion of knit semi-

* This research was supported in part by the Grant in Aid for Scientific Research, the Mintisry of Education, Science and Culture.