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PSEUDO-ANOSOV HOMEOMORPHISMS WHICH EXTEND TO ORIENTATION REVERSING HOMEOMORPHISMS OF S^3

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1. Introduction

Let F be the orientable closed surface of genus $g(>1)$, and $\{l_1, \cdots, l_g\}$ be a system of mutually disjoint, non-parallel essential loops on F suth that $\cup l_i$ cuts F into a $2g$ punctured sphere. Let f be a self-homeomorphism of F. Then $\bar{M}_f$ denotes the 3-manifold which is obtained from $F\times[0, 1]$ by attaching 2-handles along the simple loops $l_1\times\{0\}, \cdots, l_g\times\{0\}, f(l_1)\times\{1\}, \cdots, f(l_g)\times\{1\}$. We note that $\partial\bar{M}_f$ consists of two 2-spheres. Then M_f denotes the closed 3-manifold which is obtained from $\bar{M}_f$ by capping off the boundary by 3-cells. It is easy to see that if g is isotopic to f, then M_g is homeomorphic to M_f. Then, in [7], T. Yoshida posed the following question.

Question (Yoshida). *Suppose that f is a pseudo-Anosov homeomorphism. Does there exist a constant n_f such that $\pi_1(M_{f^n})\neq\{1\}$ for all $n>n_f$?*

In this note we will give a negative answer to this question.

Theorem. *For each $g(>1)$ there exist infinitely many pseudo-Anosov homeomorphisms f such that*

$$M_{f^{2n}}=\#_{i=1}^{g}(S^2\times S^1)_i\,,\quad \textit{and}$$

$$M_{f^{2n+1}}=S^3,\quad \textit{where } S^m \textit{ denotes the m-dimensional sphere.}$$

Actually we will show that there are infinitely many pseudo-Anosov homeomorphisms of Heegaard surfaces of S^3 which have the property described in the title of this note (Theorem 2.1).

In the following, we assume that the reader is familiar with [2]. For the definitions of standard terms in the 3-dimensional topology, we refer to [4].

2. Proof of Theorem

In this section we will give the proof of Theorem.