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GAP THEOREMS FOR CERTAIN SUBMANIFOLDS OF EUCLIDEAN SPACES AND HYPERBOLIC SPACE FORMS

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Introduction

Simons [18] studied minimal submanifolds of spheres and showed, among other things, that a compact minimal submanifold M of the unit n-sphere must be totally geodesic if the square length of the second fundamental form is less than $n/(2-p^{-1})$ ($p=\operatorname{codim} M$) (cf. [13] for the equality discussion). Later, Ogiue [16] and Tanno [20] considered complex submanifolds in the complex projective space and obtained similar results to the Simons' theorem (cf. [17] for other related topics and the references). On the other hand, Greene and Wu [9] have proven a gap theorem for noncompact Riemannian manifolds with a pole (cf. [7] [10] [14]). Roughly speaking, their theorem says that a Riemannian manifold with a pole whose sectional curvature goes to zero in farster than quadratic decay is isometric to Euclidean space if its dimension is greater than two and the curvature does not change its sign. These gap theorems suggest that one could expect similar results for certain open submanifolds of Euclidean space, the hyperbolic space form, the complex hyperbolic space form, etc.. Actually in this note, we shall prove the following theorems.

Theorem A.

(I) *Let M be a connected, minimal submanifold of dimension m properly immersed into Euclidean space $\boldsymbol{R}^n$. Let $\bar{\rho}$ denote the distance in R^n to a fixed point of R^n. Then M is totally geodesic if one of the following conditions holds:*

(A-i) *$m\geqq 3$, M has one end and the second fundamental form α_M of the immersion $M\to\boldsymbol{R}^n$ satisfies*

$$\limsup \bar{\rho}(x)\,|\alpha_M|(x)<\kappa_0<1\,,$$

where κ_0 is defined by $\kappa_0\{(1-\kappa_0^2)^{-1}+1\}=\sqrt{2}$.

(A-ii) *$m=2$, M has one end and*

$$\sup \bar{\rho}^2(x)\,|\alpha_M|(x)<+\infty\,.$$

(A-iii) *$2m>n$, M is imbedded and*