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ON THE SEQUENCES INDUCED FROM AUSLANDER-REITEN SEQUENCES

Dedicated to Professor Hirosi Nagao on his 60th birthday

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0. Introduction

Let kG be the group algebra of a finite group G over an algebraically closed field k of characteristic p, $p\neq 0$. Fix a normal subgroup N of G and a non-projective indecomposable kN-module V. Let SV: $0\to\Omega^2V\to X\to V\to 0$ be the Auslander-Reiten sequence terminating at V. Here Ω denotes the Heller operator. In this paper, we study the induced sequence $0\to(\Omega^2V)^G\to X^G\to V^G\to 0$. We shall decompose it according to the decomposition of V^G and investigate the relation between the sequences appearing in the decomposition and the Auslander-Reiten sequences terminating at the indecomposable direct summands of V^G. For example, we shall give a condition which guarantees that some Auslander-Reiten sequences appear in the decomposition of the induced sequence. This result is related to the work of Knörr [6].

Notation is standard. All the kG-modules considered here are finite dimensional right modules. For kG-modules W and W', we use $(W, W')^G$ to denote $\mathrm{Hom}_{kG}(W, W')$. An element f of $(W, W')^G$ is said to be projective if there are a projective kG-module P and maps $\alpha\in(W, P)^G$ and $\beta\in(P, W')^G$ such that $f=\beta\circ\alpha$. We denote by $(W, W')^{1,G}$ the factor space of $(W, W')^G$ divided by the subspace consisting of projective homomorphisms. Note that $(W, W')^{1,G}$ is an $\mathrm{End}_{kG}(W')$-$\mathrm{End}_{kG}(W)$-bimodule. For any k-algebra R, we denote its radical by JR. Unless otherwise noted, $\otimes$ means $\otimes_{kN}$.

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1. Decomposition of the induced sequence

Throughout this paper except Theorem 2.5, we deal with the situation in the Introduction. Let $E=\mathrm{End}_{kG}(V^G)$ and $E_1=\mathrm{End}_{kN}(V)$. Then E_1 can naturally be considered as a subalgebra of E by the injection ι: $E_1\to E$ defined