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ON THE THEOREMS OF GASHÜTZ AND WILLEMS

Dedicated to Professor H. Tominaga on his 60th birthday

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1. Introduction

Let p be a prime and $F=\boldsymbol{Z}/(p)$. Let G be a finite group. By *a p-chief factor*, we mean a chief factor group $V=H/K$ which is a p-group, where $H\supset K$ are normal subgroups of G. Since V is elementary, it is regarded as an irreducible right FG-module. If V has a complement in G/K, then it is called complemented. Now let us fix a chief series of G;

$$E\colon 1=G_0\subset G_1\subset\cdots\subset G_n=G\,.$$

For an irreducible FG-module U we put

$$m(E,\,U)=\;|\{i;\,G_i/G_{i-1}\simeq U \text{ and } G_i/G_{i-1} \text{ is complemented.}\}\,|$$

Let J be the radical of FG and let e be a primitive idempotent of FG such that eFG/eJ is isomorphic to the trivial FG-module F. Recently it is shown that

Theorem 1 (Willems [3]). *If V is a complemented p-chief factor of G, then it appears as a component of eJ/eJ^2 with multiplicity at least $m(E,\,V)$.*

On the other hand the following result is known as a theorem of Gashütz (see [1] Theorem 15.5 and Remark 15.6).

Theorem 2. *Suppose that G is p-solvable. Then if U is a simple component of eJ/eJ^2, U must be isomorphic to a complemented p-chief factor of G and it appears exactly $m(E,\,U)$ times in eJ/eJ^2.*

All known proofs of Gashütz's theorem involve essentially cohomological arguments and take some efforts to understand. In this short note we shall give an elementary approach to both theorems, which will be very lucid as well.

2. Preliminary Lemmas

In this section we prove several Lemmas. Some of them are possibly known, but we give proofs for the completeness. The notation used in the introduction will be fixed throughout. In addition we use $I(G)$ to denote