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STOCHASTIC CALCULUS RELATED TO NON-SYMMETRIC DIRICHLET FORMS

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0. Introduction

The theory of symmetric Dirichlet spaces and the probabilistic potential theory built on Hunt processes were unified by M. Fukushima [8], M.L. Silverstein [17] and others (see References in [8]). In particular, analysis based on additive functionals (AF's) and stochastic calculus related to symmetric Dirichlet spaces were developed by M. Fukushima [8], M. Fukushima and M. Takeda [9], S. Nakao [15] and M. Takeda [20]. On the other hand, the theory of non-symmetric Dirichlet spaces was studied by J. Bliedtner [3, 4], H. Kunita [10] etc.. Furthermore S. Carrillo Menendez [5] constructed the Hunt process associated with a non-symmetric Dirichlet space. Then many results in the symmetric case have been extended to the non-symmetric case by Y. Le Jan [11, 12], M.L. Silverstein [18], S. Carrillo Menendez [6] etc.. The purpose of this paper is to extend those results in [8], [9] and [20] to the non-symmetric case and thereby enlarge the range of applications of Dirichlet space theory.

1. Summary of the results

We first give a precise definition of non-symmetric Dirichlet form. Let X be a locally compact Hausdorff space with countable base and m a non-negative Radon measure on X such that $\text{supp}[m]=X$. $L^2(X, m)$ denotes the real L^2-space with inner product

$$(u, v)_{L^2}=\int_X u(x)\, v(x)\, m(dx)\,, \quad u, v\in L^2(X, m)\,.$$

Let $\boldsymbol{H}$ be a dense linear subspace of $L^2(X, m)$ which forms a Hilbert space with a norm $\|\ \|_{\boldsymbol{H}}$ such that for some $K>0$, $\|u\|_{\boldsymbol{H}}\geqq K\|u\|_{L^2}$ for any $u\in\boldsymbol{H}$. Moreover we assume that if $u\in\boldsymbol{H}$, then $|u|$, $u\wedge 1\in\boldsymbol{H}$. In this article we consider a bilinear form $\mathbf{a}$ on $\boldsymbol{H}\times\boldsymbol{H}$ which satisfies the following conditions;

($\mathbf{a}$.1) $\mathbf{a}_\alpha$ is coercive for any $\alpha>0$, i.e., there exists a constant $K_1=K_1(\alpha)>0$ such that $\mathbf{a}_\alpha(u, u)\geqq K_1\|u\|_{\boldsymbol{H}}^2$ for every $u\in\boldsymbol{H}$,