Asoh, T.
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ON SMOOTH $SL(2, C)$ ACTIONS ON 3-MANIFOLDS

Dedicated to Professor Masahiro Sugawara on his 60th birthday

TOHL ASOH

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1. Introduction

Real analytic actions of the special linear group $SL(n, F)$ on analytic manifolds M are classified by C.R. Schneider [6] in case that $n=2$, $F=R$ and M is any closed surface, and by F. Uchida [7–8] in case that M is the m-sphere S^m with $5\leqq n\leqq m\leqq 2n-2$ if $F=R$ and $14\leqq 2n\leqq m\leqq 4n-2$ if $F=C$.

In this paper we are concerned with smooth $SL(2, C)$-actions on closed 3-manifolds M^3. Note that $SL(2, C)$ is simple and contains $SU(2)$ as a maximal compact subgroup. Then we have the following (cf. [1; Th.1.3])

(1.1) *If $SL(2, C)$ acts non-trivially on M^3, then so does $SU(2)$ and M^3 is a quotient space of S^3 or $S^2\times S^1$; S^3/Z_n or $(S^2\times S^1)/Z_2$ (Z_n is a cyclic group of order n).*

By this reason we are concerned mainly for the case $M^3=S^3$, and we study the equivariant homeomorphism classes of such actions.

In case of transitive actions, we see the following

Theorem 1.2. *There are real analytic $SL(2, C)$-actions ϕ_r on S^3 for $r\in R$, which are not equivariantly homeomorphic to each other, and any transitive $SL(2, C)$-action on S^3 is equivariantly diffeomorphic to some ϕ_r (see (4.1) for the definition of ϕ_r).*

In case of non-transitive actions, the classification of $SL(2, C)$-action on S^3 can be reduced to that of pairs of a one-parameter transformation group on $S^1(\subset C)$ and a real valued smooth function on $S^1-\{\pm 1\}$; and further to that of triads of subsets A and B_i ($i=1, 2$) of S^1 satisfying

(1.3) (A1) *$A(\neq\phi)$ is a finite union of closed intervals, $A\cap J(A)=\phi$ and the components of A alternate with those of $J(A)$, where J is the reflections on S^1 in the real line.*
(A2) *$B_i(i=1, 2)$ are open in S^1 and $B_1\cup B_2\subset A-\partial A$.*

Such triads (A, B_i) and (A', B_i') are called A-*equivalent if there is an orientation preserving homeomorphis Φ of S^1 onto itself such that $\Phi J=J\Phi$ and*