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HOMOTOPY REPRESENTATION GROUPS AND SWAN SUBGROUPS

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0. Introduction

Let G be a finite group. A finite dimensional G-CW-complex X is called a homotopy representation of G if the H-fixed point set X^H is homotopy equivalent to a ($\dim X^H$)-dimensional sphere or the empty set for each subgroup H of G. Moreover if X is G-homotopy equivalent to a finite G-CW-complex, then X is called a finite homotopy representation of G and if X is G-homotopy equivalent to a unit sphere of a real representation of G, then X is called a linear homotopy representation of G. T. tom Dieck and T. Petrie defined homotopy representation groups in order to study homotopy representations. Let $V^+(G, h^\infty)$ be the set of G-homotopy types of homotopy representations. We define the addition on $V^+(G, h^\infty)$ by the join and so $V^+(G, h^\infty)$ becomes a semigroup. The Grothendieck group of $V^+(G, h^\infty)$ is denoted by $V(G, h^\infty)$ and called the homotopy representation group. A similar group $V(G, h)$ [resp. $V(G, l)$] can be defined for finite [resp. linear] homotopy representations.

Let $\phi(G)$ denote the set of conjugacy classes of subgroups of G and $C(G)$ the ring of functions from $\phi(G)$ to integers. For a homotopy representation X, the dimension function $\operatorname{Dim} X$ in $C(G)$ is defined by $(\operatorname{Dim} X)(H)=\dim X^H+1$. (If X^H is empty, then we set $\dim X^H=-1$.) Then

$$\operatorname{Dim} X*Y = \operatorname{Dim} X+\operatorname{Dim} Y$$

for any two homotopy representations. ("*" means the join.) Hence one can define the homomorphism

$$\operatorname{Dim}\colon V(G, \lambda) \to C(G) \quad (\lambda = h^\infty, h \text{ or } l)$$

by the natural way. The kernel of Dim is denoted by $v(G, \lambda)$. tom Dieck and Petrie proved that $v(G, \lambda)$ is the torsion group of $V(G, \lambda)$ and

$$v(G, h^\infty) \cong \operatorname{Pic} \Omega(G)\,, \qquad (0.1)$$

where $\operatorname{Pic} \Omega(G)$ is the Picard group of the Burnside ring $\Omega(G)$.

There are the natural homomorphisms