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STRUCTURES OF FULL HAKEN MANIFOLDS

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1. Introduction

In this paper, we consider some relations between a Heegaard splitting and the torus decomposition of a Haken manifold. The first result of this paper is:

Theorem 1. *Let M be a Haken manifold without boundary or with incompressible toral boundary. Suppose that M admits a Heegaard splitting of genus* $g(\geq 2)$. *Then M is decomposed into at most* $3g$-3 *components by the torus decomposition. Moreover, if M is decomposed into* $3g$-3 *components, then each component is simple i.e. every incompressible torus in it is boundary parallel.*

For the definition of a Heegaard splitting, and the torus decomposition of a 3-manifold with boundary in this context, see section 2.

The classical Haken's theorem ([H], [J]) shows that a Heegaard genus g 3-manifold is decomposed into at most g components by the prime decomposition. Theorem 1 is an analogy to this fact.

REMARK. We note that the above estimation is best possible. In section 8, we will show that for each $g(\geq 2)$ there are infinitely many Haken manifolds with Heegaard splittings of genus g, each of which is decomposed into $3g$-3 components by the torus decomposition.

The key of the proof of Theorem 1 is Proposition 4.1, which is an analogy to the Haken's theorem.

Proposition 4.1. *Let M be a Haken manifold as in Theorem* 1, *and* $\mathcal{T}$ *be a union of tori which gives the torus decomposition of M. If the number of the components of* $\mathcal{T}$ *is greater than or equal to* $3g$-4, *then there is a component T of* $\mathcal{T}$ *such that T is ambient isotopic to T′ which intersects the genus g Heegaard surface in a circle.*

Let M be a Haken manifold as in Theorem 1. We say that M is *full* if it

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