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YANG-MILLS CONNECTIONS AND MODULI SPACE

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0. Introduction

There are many researches on deformations of Yang-Mills connections over 4-dimensional manifolds. In this paper, we generalize the results into higher dimensional cases. In 4-dimensional case, the Hodge $*$-operator acts on $\Lambda^2 M$ and the notion of (anti-)self dual connection is introduced, which brings beautiful results in Atiyah, Singer and Hitchin [1]. Therefore, in higher dimensional cases, we have to assume some properties on the base riemannian manifold. In [1], it is already pointed out that if M is a (2-dimensional) complex manifold, an anti-self dual connection defines a holomorphic structure of the bundle. Itoh [6] considers in full this situation, which we will generalize by the notion of "Einstein holomorphic connection" over a Kähler manifold. However, the moduli space of Yang-Mills connections over a higher dimensional Kähler manifold may have many singularities, and probably we can not expect that the moduli space becomes a manifold.

The fundamental notions in this paper come from [1], and fundamental idea comes from Koiso [9]. It is remarkable that the results for the moduli space of Einstein metrics and that of Yang-Mills connections are quite analogous. In fact we will get the following results.

Theorem 2.7 (c.f. [9, Theorem 3.1]). *The local pre-moduli space is a finite dimensional real analytic set.*

Corollary 6.5 (c.f. [9, Theorem 10.5]). *If the initial structure* (*Einstein metric or Yang-Mills connection*) *is compatible with a complex structure, then also around structures are compatible with some complex structures.*

Theorem 9.3 (c.f. [9, Theorem 12.3]). *Under some assumption, the local pre-moduli space has a canonical Kähler structure.*

However, there is an important difference. For Einstein metrics, we have no effective obstruction spaces for deformation ([9, Proposition 5.4]), but for Yang-Mills connections we have one (Theorem 6.9).