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GENERALIZATIONS OF NAKAYAMA RING IV

(LEFT SERIAL RINGS WITH (*, I))

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Let R be an algebra over an algebraically closed field K with finite dimension. Under an assumption $J^4=0$, we have studied a left serial algebra with (*, 1): the radical of any hollow right R-module is always a direct sum of hollow modules, in [3], where J is the Jacobson radical of R. In this case eJ/eJ^2 is square-free, i.e., a direct sum of simple modules, which are not isomorphic to one another. We shall give, in this note, a complete characterization of a left serial ring with (*, 1) under the assumption: eJ/eJ^2 square-free. In the forthcoming paper [5], we shall study a left serial ring with (*, 1) in general.

1. Definitions and preliminaries

In this note we only deal with a left and right artinian ring R with identity. We assume that every R-module M is a unitary right (or left) R-module and denote its Jacobson radical and socle by $J(M)$ and Soc(M), respectively. $|M|$ means the length of a composition series of M. If M has a unique composition series, we call M a *uniserial module*. If, for each primitive idempotent e, eR is uniserial as a right R-module, we call R a *right serial ring* (*Nakayama ring*).

We obtained a characterization of a right serial ring in terms of submodules in a direct sum of uniserial modules [1]. As a generalization of the above result, we studied the following property:

(*, n) *Every maximal submodule of a direct sum of n hollow modules is also a direct sum of hollow modules* [2].

In this note we shall study a ring with (*, 1), i.e., every factor module of eJ is a direct sum of hollow modules for each primitive idempotent e, where $J=\mathrm{J}(R)$. Concerning (*, 1) we got

Lemma A ([4], Theorem 4). *Let R be a right artinian ring. Then R satisfies* (*, 1) *for any hollow right R-module if and only if the following two conditions are fulfiled:*

1) $eJ=\sum_{i=1}^{m}\oplus A_i$, *where e is any primitive idempotent in R and the A_i are hollow.*