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# ON SOME EXTENSION OF 1-SPREAD SETS

Dedicated to Professor Hirosi Nagao on his 60th birthday

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## 1. Introduction

A set $\Sigma$ of $q^2$ (2,2)-matrices over $K=GF(q)$ is said to be *a 1-spread set* if it contains the zero matrix 0 and $X$-$Y$ is nonsingular for any distinct $X$, $Y\in\Sigma$. Let $\Sigma'$ be an arbitrary 1-spread set over $K$. Then $\Sigma'=\{\begin{pmatrix} x & y \\ g(x,y) & h(x,y)\end{pmatrix} | x, y \in K\}$ for suitable mappings $g$ and $h$ from $K\times K$ to $K$. Let $F=GF(q^2)\supset K$. If Char $K$, the characteristic of $K$, is odd, we can take an element $t\in F$-$K$ with $t^2\in K$ and define a mapping $f$ from $F$ to itself in such a way that $f(x+yt)=g(x,y)-h(x,y)t$ for $x, y\in K$. Then $f$ satisfies the condition

(*) $f(0)=0$ and $(x-y)(f(x)-f(y))\notin K$ for any distinct $x, y\in F$.

Furthermore the set of (2,2)-matrices

(**) $\Sigma_f=\{\begin{pmatrix} x & y \\ f(y) & x^q\end{pmatrix} | x, y\in F\}$

is a 1-spread set over $F$ and the resulting translation plane of order $q^4$ with the kernel $F$, say $\pi$, has the following properties:

(A1) The linear translation complement $LC(\pi)$ has a shears group $P$ of order at least $q^2$.

(A2) $LC(\pi)$ has a Baer subgroup $Q$ of order $q+1$ with $[P, Q]\neq 1$.

In this paper we study a class of translation planes of order $q^4$ with the properties (A1) and (A2) as above. Let $\Omega(F)$ be the set of mappings from $F$ to itself satisfying (*). Then the set of (2,2)-matrices $\Sigma_f$ defined by (**) is a 1-spread set for any $f\in\Omega(F)$ and if Char $K$ is odd, a 1-spread set $\Sigma'_f$ over $K$ corresponding to $f$ is naturally defined (Proposition 2.1). Denote by $\Pi(F)$ the set of planes $\pi_f$ corresponding to $\Sigma_f$ with $f\in\Omega(F)$. Then $\Pi(F)$ is characterized as the set of translation planes with the kernel $F$ having the properties (A1) and (A2).

The translation complements of these planes are solvable when $p>2$. To show this we need a result on shears groups (Theorem 3.1). Any of these