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PSEUDO-RANK FUNCTIONS ON SKEW GROUP RINGS AND ON FIXED SUBRINGS OF AUTOMORPHISMS OF UNIT-REGULAR RINGS

Dedicated to Professor Hisao Tominaga on his 60th birthday

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Let R be a unit-regular ring and G a finite subgroup of $Aut(R)$ with $|G|^{-1} \in R$. This paper is concerned with relationships between the pseudo-rank functions of the skew group ring $R*G$ and ones of the fixed subring R^G. We introduce such relationships by studying certain homomorphisms between $K_0(R*G)$ and $K_0(R^G)$.

In §1, under the assumption that $R*G$ is a unit-regular ring and R is a finitely generated projective left R^G-module, we shall investigate the following two homomorphisms:

$$\bar{\mu}: K_0(R^G) \to K_0(R*G)\,, \quad \text{defined by} \quad \bar{\mu}([M]) = [R*Ge \otimes_{R^G} M]$$

$$\bar{\lambda}: K_0(R*G) \to K_0(R^G)\,, \quad \text{defined by} \quad \bar{\lambda}([A]) = [\mathrm{Hom}_{R*G}(R*Ge, A)]\,,$$

where $e = |G|^{-1} \sum_{g \in G} g$ in $R*G$. Then we shall show that $\bar{\lambda}\,\bar{\mu}$ is the identity map and $\bar{\mu}$ is an order-embedding map.

The maps $\bar{\mu}$, $\bar{\lambda}$ induce maps μ^*, λ^* between $\mathrm{P}(R*G)$ and $\mathrm{P}(R^G)$, where $\mathrm{P}(T)$ (resp. $\partial_e\mathrm{P}(T)$) is the family of all pseudo-rank functions (resp. extremal pseudo-rank functions) of a regular ring T. For any $N \in \mathrm{P}(R*G)$ with $N(e) > 0$ and any $a \in R^G$, we define

$$\mu^*(N)\,(a) = N(e)^{-1}\, D_N(R*Ge \otimes_{R^G} R^G a)\,,$$

where D_N is the dimension function which corresponds to N. For any $Q \in P(R^G)$ and any $x \in R*G$, we define

$$\lambda^*(Q)\,(x) = D_Q({}_{R^G}R)^{-1} D_Q(\mathrm{Hom}_{R*G}(R*Ge, R*Gx))\,,$$

where D_Q is the dimension function which corresponds to Q. Then we shall show that $\mu^*(N)$ (resp. $\lambda^*(Q)$) is a pseudo-rank function of R^G (resp. $R*G$) and $\mu^*\lambda^*$=identity and μ^* preserves extremal pseudo-rank functions.

In §2, for a directly finite, left self-injective, regular ring R and an X-