Shigekawa, I.
Osaka J. Math.
24 (1987), 37-59

EXISTENCE OF INVARIANT MEASURES OF DIFFUSIONS ON AN ABSTRACT WIENER SPACE

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(Received November 25, 1985)

1. Introduction

In this paper, we consider diffusions on an *abstract Wiener space* (B, H, μ), B is a separable (real) Banach space with a norm $\|\cdot\|_B$, H is a separable (real) Hilbert space that is densely and continuously imbedded in B with an inner product $\langle\cdot, \cdot\rangle_H$ and a norm $|\cdot|_H=\sqrt{\langle\cdot, \cdot\rangle_H}$ and μ is the Wiener measure, i.e., Borel probability measure with the characteristic function $\hat{\mu}$ given by

$$\hat{\mu}(l) = \int_B e^{\sqrt{-1}(x,l)}\,\mu(dx) = \exp\{-\frac{1}{2}|l|_H^2\}, \qquad l\in B^*$$

where B^* is the dual space of B, $(\ ,\ )$ is the natural bilinear form on $B\times B^*$ and we regard B^* as a subspace of H: $B^*\subseteq H^*=H$.

Typical example of a diffusion on the abstract Wiener space is the *Ornstein-Uhlenbeck process*. We denote its generator by $\frac{1}{2}L$ and call L the Ornstein-Uhlenbeck operator. We consider diffusions generated by operators of the form $A=\frac{1}{2}L+b$ where b is an H-valued bounded function on B and we regard b as a vector field on B. Our main aim is to show the *existence of invariant measures* of these diffusions.

By the way, as is well-known, such a diffusion is obtained by the transformation of the drift for the Ornstein-Uhlenbeck process. Hence our diffusions are closely related to the Ornstein-Uhlenbeck process. But, a calculus for the Ornstein-Uhlenbeck process, sometimes called *Malliavin's calculus*, was developed by many authors. So our discussion is based on Malliavin's calculus, especially on the theories of Ornstein-Uhlenbeck semigroup and Sobolev spaces over the abstract Wiener space which were studied by P.A. Meyer and H. Sugita. In this paper, we mainly follow Sugita [10].

Our strategy to prove the existence of an invariant measure is to solve the equation $A^*\rho=0$ where A^* is the dual operator of A. First we solve this equation in finite dimensional case by using the stability of the index. Secondly we solve it in infinite dimensional case by limiting procedure. In the second