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Osaka J. Math.
23 (1986), 881-927

ACTIONS OF COMPACT LIE GROUPS AND THE EQUIVARIANT WHITEHEAD GROUP

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(Received August 20, 1985)

1. Introduction. Let G be a compact Lie group and X a finite G-CW complex. By $Wh_G(X)$ we denote the equivariant Whitehead group of X as defined in [5]. The group $Wh_G(X)$ is defined in a geometric way and $Wh_G(X)$ is an abelian group. If $f\colon X\to Y$ is a G-homotopy equivalence between finite G-CW complexes then the (geometric) equivariant Whitehead torsion of f is an element $\tau(f)\in Wh_G(X)$ and f is a simple G-homotopy equivalence if and only if $\tau(f)=0$, see Theorem II.3.6′ in [5].

Let us here first state the two main results of this paper and then describe some earlier work on the subject, and also give a quick outline of some other results and constructions contained in this paper which are of independent interest. The first main result has to do with the algebraic determination of $Wh_G(X)$.

Theorem A. *There exists an isomorphism*

$$\Phi\colon Wh_G(X)\xrightarrow{\cong}\sum_{C(X)}\oplus Wh(\pi_0(WH)^*_\alpha)\,.$$

In the above formula the right hand side is a direct sum of ordinary (algebraically) defined Whitehead groups of discrete groups $\pi_0(WH)^*_\alpha$ which will be defined below. The direct sum is over the set $C(X)$ of equivalence classes of connected components X^H_α of arbitrary fixed point sets X^H, where H is any closed subgroup of G. The components X^H_α and X^K_β of the fixed point sets X^H and X^K, respectively, are defined to be in relation, denoted

$$X^H_\alpha\sim X^K_\beta$$

if there exists $n\in G$ such that $nHn^{-1}=K$ and $n(X^H_\alpha)=X^K_\beta$. Given a component X^H_α of X^H we define

$$(WH)_\alpha=\{w\in WH\,|\,wX^H=X^H\}\,.$$

Here $WH=NH/H$, and NH denotes the normalizer of H in G. The group $(WH)^*_\alpha$ is a Lie group (not necessarily compact) which acts on the universal