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JORDAN-HÖLDER THEOREM FOR PSEUDO-SYMMETRIC SETS

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1. Introduction

A pseudo-symmetric set is a pair (U, σ) where U is a set and σ is a mapping of U into the group of permutations on U such that $\sigma(u)$ fixes u for every element u in U and that it satisfies a fundamental identity: $\sigma(u^{\sigma(v)})=\sigma(v)^{-1}\sigma(u)\sigma(v)$ for u and v in U.

In [1], a possibility of developing a structure theory of pseudo-symmetric set is indicated. In this paper, we shall establish an analogue of Jordan-Hölder theorem in group theory for pseudo-symmetric sets.

Contrary to group theory, the concept of kernels of homomorphisms is not available. Instead, a concept of a normal decomposition is introduced in [1]. It is a partition of U such that each class of the partition consists of elements that are mapped to an element by a given homomorphism. When a partition A is a refinement of a partition B, we denote $A\leq B$. The partition of U which has just one class U itself is denoted by U. The complete partition of U whose classes are one-point sets is denoted by E. So, $E\leq A\leq U$ for every partition A. Suppose we have a sequence of normal decompositions P_i such that

$$(1) \qquad U=P_0>P_1>P_2>\cdots>P_n=E$$

where there is no normal decomposition between P_i and P_{i+1}. Suppose we have another sequence of normal decompositions Q_j of the same properties:

$$(2) \qquad U=Q_0>Q_1>Q_2>\cdots>Q_m=E\,.$$

We say that P_i/P_{i+1} is non-trivial if $H(P_i/P_{i+1})\neq 1$, where $H(P_i/P_{i+1})$ is the group of displacements for P_i/P_{i+1}. (The definition will be given in **3.**) The main theorem we obtain is that between the set of non-trivial P_i/P_{i+1} and that of non-trivial Q_j/Q_{j+1} there is a one to one correspondence such that if P_i/P_{i+1} corresponds to Q_j/Q_{j+1} then $H(P_i/P_{i+1})\simeq H(Q_j/Q_{j+1})$.

2. Partitions of a set

Let U be a (universal) set, and $U=\cup A_i$ a partition of U into non-empty