Gyoja, A.
Osaka J. Math.
23 (1986), 841-852

A q-ANALOGUE OF YOUNG SYMMETRIZER*

AKIHIKO GYOJA

(Received August 9, 1985)

Let W be the symmetric group on the set of n letters $\{1, 2, \cdots, n\}$, s_i $(1 \leqq i \leqq n-1)$ the transposition $(i, i+1)$ in W, and $S=\{s_1, s_2, \cdots, s_{n-1}\}$. Then every element w of W can be expressed as $w=s_{i_1}s_{i_2}\cdots s_{i_l}$ $(1 \leqq i_\alpha \leqq n-1)$. We denote the minimal length of such an expression by $l(w)$, i.e., $l(w)=\min\{l\}$. Let $K=\boldsymbol{C}(q)$ be the field of rational functions in one variable q over the complex number field $\boldsymbol{C}$. The Hecke algebra $H=H(q)$ of W is defined as follows: H has a basis $\{h(w)\}_{w\in W}$ which is parametrized by the elements of W. The multiplication is characterized by the rules

$$(h(s)+1)(h(s)-q) = 0\,, \qquad \text{if} \quad s\in S\,,$$
$$h(w)h(w') = h(ww')\,, \qquad \text{if} \quad l(w)+l(w') = l(ww')\,.$$

Notice that H is a q-analogue of the group algebra $\boldsymbol{C}W$ of W in the sense that when q is specialized to 1, H is specialized to $\boldsymbol{C}W$. It should also be mentioned that the Hecke algebra can be defined for a general Coxeter system (W, S) (see [2; Chap. 4, §2, Ex. 23]).

As is well-known, a complete set of mutually orthogonal primitive idempotents of $\boldsymbol{C}W$ is constructed by A. Young (see, for example, [6], [9]). Our main theorems are (3.10) and (4.5). In these theorems, we give a complete family of mutually orthogonal primitive idempotents of H, which is specialized to the one constructed by Young, when q is specialized to 1.

The present work was motivated by a question posed by Dr. M. Jimbo in connection with his investigation [7] of the Yang-Baxter equation in mathematical physics. The author would like to express his thanks to Dr. M. Jimbo.

1. Let (W, S) be a Coxeter system, w an element of W and $w=s_1s_2\cdots s_n$ $(s_i\in S)$ a reduced decomposition of w. See [2; Chap. IV] for the fundamental concepts concerning Coxeter systems. It is known and easily proved by using [2; Chap. IV, n° 1.5, Lemma 4] that the set

$$\{s_{i_1}s_{i_2}\cdots s_{i_p} \mid 1\leqq i_1<\cdots<i_p\leqq n,\ 0\leqq p\leqq n\}$$

* This research was supported in part by Grant-in-Aid for Scientific Research, The Ministry of Education, Science and Culture.