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ON MAXIMAL SUBMODULES OF A FINITE DIRECT SUM OF HOLLOW MODULES V

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Throughout this note, R is a ring with identity, J the Jacobson radical of R, and all R-modules are unitary right R-modules. Further we let e be a local idempotent of R and U_R a submodule of eJ such that eR/U is of finite length. Put $D:=eRe/eJe$ and $D(U):=\{x+eJe\in D \mid x\in eRe,\ xU\leq U\}$. Then $D(U)$ is a division subring of D. In [3]-[6], we have given a relationship between the dimension $[D: D(U)]_r$ of D as a *right* $D(U)$-vector space and the property (**, n) of maximal submodules of $(eR/U)^{(n)}$ defined there. (The dual result had been obtained in [1, Proposition 2.1] from another point of view.)

In this short note, we shall study the dimension $[D: D(U)]_l$ of D as a *left* $D(U)$-vector space and give it a meaning. Originally our considerations had been restricted to the case of uniform modules of Loewy length 2 over an artinian ring with proofs along the line of Sumioka [8, Lemma 5.3], and later by different proofs we generalized and dualized to get the present form. Hence it should be noted that by dualizing the arguments all the parallel results hold for uniform modules if we assume that $[D: D(U)]_l<\infty$.

In what follows, we denote by $|M|$ and by $\sharp I$ the composition length of each R-module M and the cardinality of each set I, respectively. Let L be a submodule of an R-module M. Then we say that L is a *characteristic* submodule of M if $fL\leq L$ for every endomorphism f of M. By $M^{(I)}$ we denote the direct sum of $\sharp I$ copies of M for each R-module M and each set I. We regard $Me=\mathrm{Hom}_R(eR, M)$ for every R-module M by identifying each $t\in Me$ with the map $eR\to M$ defined via $x\mapsto tx$ for each $x\in eR$. So in particular for each $t\in eRe=\mathrm{End}_R(eR)$, $t^{-1}U$ means the inverse image $\{x\in eR \mid tx\in U\}$ of U under t.

Now the canonical epimorphism π: $eR\to eR/U$ induces a monomorphism $\mathrm{End}_R(eR/U)\to\mathrm{Hom}_R(eR, eR/U)$ by which we regard $\mathrm{End}_R(eR/U)=\{f\in\mathrm{Hom}_R(eR, eR/U) \mid f(U)=0\}\leq\mathrm{Hom}_R(eR, eR/U)$. Consider the epimorphism

$$\delta\colon \mathrm{Hom}_R(eR, eR/U) \to \mathrm{Hom}_R(eR, eR/eJ) = eRe/eJe$$

induced from the canonical epimorphism $eR/Ue\to R/eJ$ by the exact functor