Nakamoto, T.
Osaka J. Math.
23 (1986), 823-833

PRIME ONE-SIDED IDEALS OF A FINITE NORMALIZING EXTENSION

Dedicated to Professor Hisao Tominaga on his 60th birthday

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(Received November 30, 1984)
(Revised November 27, 1985)

Introduction

Throughout the present paper, R will represent a ring with identity 1. Let I be a right ideal of R, and $b_R(I)=\{r\in R\mid Rr\subset I\}$. Then, $b_R(I)$ is the largest ideal of R contained in I. We shall call that I is a *prime right ideal* provided that if X and Y are right ideals of R with $XY\subset I$, then either $X\subset I$ or $Y\subset I$. It is clear that a maximal right ideal is a prime right ideal. If I is a prime right ideal, then $b_R(I)$ is a prime ideal. Next, let S be a ring extension of a ring R with the same identity 1. S is said to be a *left torsionfree* R-bimodule if $r_S(X)=0$ for every essential ideal X of R, where $r_S(X)$ is the right annihilator of X in S (cf. [1]). Right torsionfree is defined similarly, and S is said to be *torsionfree* if it is both left and right torsionfree. Moreover, S is said to be *fully torsionfree* if, for every prime ideal P of S, S/P is a right torsionfree $R/(P\cap R)$-bimodule (cf. [3]). Furthermore, we say that S is a *finite normalizing extension* (resp. a *liberal extension*) of R if there exists a finite subset $\{a_1, a_2, \cdots, a_n\}$ of S such that $S=\sum_{i=1}^n Ra_i$ and $Ra_i=a_iR$ for all $i=1, 2, \cdots, n$ (resp. $ra_i=a_ir$ for all $r\in R$ and for all $i=1, 2, \cdots, n$). A ring extension T of R is said to be an *intermediate normalizing extension* (resp. an *intermediate extension*) if there exists a finite normalizing extension (resp. a liberal extension) S of R containing T.

Recently, Heinicke and Robson [1, 2], Lorenz [5], Jabbour [3] and others, gave some descriptions of the relationship between the prime ideals of R and any intermediate normalizing extension T. In this paper, we shall verify that there is a similar relationship between the prime right ideals of R and T. In Section 1, we shall prove a "lying over" theorem for a liberal extension, and a "lying inside" theorem and a "lying outside" theorem for an intermediate extension. In Sections 2 and 3, we shall prove a "cutting down" theorem for a fully torsionfree finite normalizing extension and an intermediate normalizing extension of a fully torsionfree finite normalizing extension.