Asaba, T.
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ASYMPTOTIC DIRICHLET PROBLEM FOR A COMPLEX MONGE-AMPÈRE OPERATOR

TAKASHI ASABA

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1. Introduction

Let M be a complex manifold of dimension n and $P(M)$ denote the set of plurisubharmonic functions on M. For $u\in P(M)\cap C^2(M)$, we write $(dd^c u)^k$ for $\underbrace{dd^c u\wedge dd^c u\wedge\cdots dd^c u}_{k \text{ times}}$ where $d^c=\sqrt{-1}(\bar\partial-\partial)$. In the case $k=n$, the operator $u\to(dd^c u)^n$ is called a complex Monge-Ampére operator. In general, let u be a locally bounded plurisubharmonic function on M. In [5], [6], Bedford and Taylor defined a positive (k, k) current $(dd^c u)^k$ inductively by

$$\int \psi\wedge(dd^c u)^k = \int u\cdot dd^c\psi\wedge(dd^c u)^{k-1}$$

for any smooth $(n-k, n-k)$ form ψ with compact support on M. In the same paper they studied the Dirichlet problem for the complex Monge-Ampére operator on strongly pseudoconvex bounded domains in $\boldsymbol{C}^n$.

In this paper we shall consider the Dirichlet problem at *infinity* on certain negatively curved Kähler manifolds. Before stating our main theorem, we recall some definitions in [10]: Let M be a simply connected complete Riemannian manifold of nonpositive curvature. Two geodesic rays γ_1, γ_2 parametrized by arc length are called *asymptotic* if the distance $d(\gamma_1(t), \gamma_2(t))$ is bounded for $t\geqq 0$. The equivalence classes of geodesic rays are called *asymptotic classes*, the set of which will be denoted by $M(\infty)$. Then $\bar M=M\cup M(\infty)$ equipped with the "cone topology" is a compact topological space homeomorphic to a cell.

Theorem. *Let M be a simply connected complete Kähler manifold whose sectional curvature K satisfies*

$$(1) \qquad -a^2\leqq K\leqq -1 \qquad (a\geqq 1)\,.$$

We denote by ω the Kähler form on M and by $r(x)$ the distance function relative to a fixed point $o\in M$. Then for any continuous function f on $M(\infty)$ and for any