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PROPAGATION OF WAVE FRONT SETS OF SOLUTIONS OF THE CAUCHY PROBLEM FOR HYPERBOLIC EQUATIONS IN GEVREY CLASSES

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Introduction. In the recent paper [18] the second author has constructed the fundamental solution of the Cauchy problem for hyperbolic equations in Gevrey classes, and investigated the propagation of wave front sets of their solutions in Gevrey classes by assuming the constant multiplicities of their characteristic roots. The purpose of the present paper is to study the propagation of wave front sets in Gevrey classes for solutions of hyperbolic equations with characteristic roots of variable multiplicities and to give a similar result to the one for the C^∞-case obtained by Kumano-go and the second author [10]. Main results of the present paper are announced in [15] and [19].

Let $\mathcal{L}$ be an $l\times l$ hyperbolic system of the form

$$(1)\qquad \mathcal{L}=D_t-\begin{bmatrix}\lambda_1(t, X, D_x) & & 0\\ & \ddots & \\ 0 & & \lambda_l(t, X, D_x)\end{bmatrix}+(b_{jk}(t, X, D_x)) \quad \text{on} \quad [0, T]\times R^n_x$$

with real symbols $\lambda_j(t, x, \xi)$ in $G^{(\kappa)}([0, T]; S^1_{G(\kappa)})$ and symbols $b_{jk}(t, x, \xi)$ in $G^{(\kappa)}([0, T]; S^\sigma_{G(\kappa)})$ $(0\leqq\sigma<1/\kappa)$. Here, for $\kappa>1$ and a real m we denote by $G^{(\kappa)}([0, T]; S^m_{G(\kappa)})$ a class of symbols $p(t, x, \xi)$ of pseudo-differential operators satisfying for any multi-indices α, β and non-negative integer γ

$$(2)\qquad |\partial_t^\gamma \partial_\xi^\alpha \partial_x^\beta p(t, x, \xi)|\leqq CM^{-(|\alpha|+|\beta|+\gamma)}(\alpha!\,\beta!\,\gamma!)^\kappa\langle\xi\rangle^{m-|\alpha|} \quad \textit{for} \quad (t, x, \xi)\in[0, T]\times R^n_x\times R^n_\xi\,,$$

with constants C and M (>0) independent of α, β and γ. Throughout the present paper we assume the symbols λ_j are positively homogeneous in ξ (for $|\xi|\geqq 1$), that is, λ_j satisfy

$$\lambda_j(t, x, \theta\xi)=\theta\lambda_j(t, x, \xi) \qquad \textit{for} \quad \theta\geqq 1 \quad \textit{and} \quad |\xi|\geqq 1\,.$$

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