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THE NINOMIYA OPERATORS AND THE GENERALIZED DIRICHLET PROBLEM IN POTENTIAL THEORY

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Introduction

Investigating various aspects of Keldyš theorem [11], N. Ninomiya introduced and studied special operators related to the generalized Dirichlet problem in classical potential theory.

To recall his interesting result from [18], let us introduce the following notation.

Let $\{B_j\}$ be the sequence of balls in the Euclidean space $\boldsymbol{R}^m$ of dimension $m>2$ having a rational center and rational radius. Denote by λ_j the normalized surface measure on ∂B_j and fix positive numbers a_j such that the Newtonian potential q of the measure $\sum a_j \lambda_j$ is continuous on $\boldsymbol{R}^m$. (The potential q will be called the Cartan potential here.)

Suppose that $U\subset\boldsymbol{R}^m$ is a bounded open set and denote by $C(\partial U)$ the set of all continuous functions on ∂U. Let $\mathcal{F}(U)$ stand for the set of all real-valued functions defined on U. As usual, H^U denotes the operator of *the Perron-Wiener-Brelot solution of the generalized Dirichlet problem on U*.

To state the Ninomiya uniqueness result, suppose that $A: C(\partial U)\rightarrow\mathcal{F}(U)$ is an operator having the following properties:

(i) A is linear and positive;

(ii) $\sup Af(U)\leqq\sup f(\partial U)$ whenever $f\in C(\partial U)$;

(iii) $A(p_{|\partial U})=p_{|U}$ whenever p is a continuous Newtonian potential of a measure supported by the complement CU of the set U;

(iv) $A(q_{|\partial U})$ is harmonic (or subharmonic) on U for the Cartan potential q.

(Obviously, the operator $A=H^U$ enjoys (i)-(iv), thus no existence problem arises.)

N. Ninomiya [18] was able to prove that such an operator A is uniquely determined by conditions (i)-(iv). This remarkable result improves the statement of Keldyš uniqueness theorem for the generalized Dirichlet problem (see Theorem 1 below; note that conditions (4) and (6) stated there are automatically satisfied in classical potential theory; cf. also [6] and [16]).

The proof of uniqueness given by Ninomiya makes use of potentials of