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SIDE-EFFECTS OF MEASURE REPRESENTATIONS ON AXIOM (D)

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The two most important examples of axiomatic potential theory are the harmonic space $(\boldsymbol{R}^n, \mathcal{H}^*_\Delta)$ of the solutions of the Laplace equation on open subsets of $\boldsymbol{R}^n$ and the space $(\boldsymbol{R}^{n+1}, \mathcal{H}^*_\Omega)$ of solutions of the heat equation on open subsets of $\boldsymbol{R}^{n+1}$. The sheaves of solutions of these two partial differential equations—and of a large class of other elliptic and parabolic differential equations—behave similarly in many respects, which led to the development of a common theory—the theory of harmonic spaces. In this paper we want to focus on two properties with respect to which the Laplace operator Δ and the heat operator $\Omega=\Delta-\frac{\partial}{\partial t}$ differ:

1) The Laplace operator Δ satisfies the *bounded energy principle*

$$\text{(E)} \quad \int\int G_\Delta(x, y)\, \mu(dx)\, \mu(dy) \geq 0$$

for every signed measure μ such that the potential

$$x \mapsto \int G_\Delta(x, y) |\mu|(dy)$$

is bounded, where G_Δ denotes the Newtonian kernel; on the ccntiary the heat operator Ω and its kernel G_Ω do not satisfy (E).

2) The Laplace operator Δ is a *fine local operator,* in contrast with Ω which is not (for the definition of fine local operators and the proof of this statement see §1). It turns out that it is Axiom (D), introduced in axiomatic potential theory by M. Brelot, which is responsible for the "fine local" property as well as for the bounded energy principle. The purpose of this paper is to formulate and prove this statement within the framework of the theory of harmonic spaces in the sense of [2] or [5].

In the theory of harmonic spaces the starting-point is a sheaf of functions—called harmonic or hyperharmonic functions—without the intervention of a defining operator. In some situations however it is useful or even necessary to have such a defining operator at one's disposal—a substitute for the differential