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ON HASSE-SCHMIDT HIGHER DERIVATIONS

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Let k be a field of characteristic zero and let A be a commutative k-algebra. A higher derivation D of A over k is a sequence

$$\underline{D} = \{D_0, D_1, D_2, \cdots\}$$

of additive k-endomorphisms D_i's such that D_0 is the identity map of A and $D_n(ab) = \sum_{m=0}^{n} D_m(a) D_{n-m}(b)$ for every $a, b \in A$. This interesting notion of higher derivations was introduced by H. Hasse and F.K. Schmidt in [1].

In this paper we shall prove that a higher derivation $\underline{D}$ of A over k is represented uniquely by a certain sequence of derivations of A over k.

Let n, r be positive integers such that $n \geq r$. We shall denote by $P_{n,r}$ the set of ordered partitions of n into r-positive integers, i.e.,

$$P_{n,r} = \{(n_1, \cdots, n_r) \mid \sum_{i=1}^{r} n_i = n, n_i \in N_+\} .$$

It is easily seen that the cardinality $|P_{n,r}|$ of the set $P_{n,r}$ is given by

$$|P_{n,r}| = \binom{n-1}{r-1} .$$

Proposition 1. *Let $\underline{D} = (D_0, D_1, D_2, \cdots)$ be a higher derivation on A and let $\delta_n (n \geq 1)$ be defined by the equations*

$$\delta_n = \sum_{r=1}^{n} \frac{(-1)^{r+1}}{r} \sum_{(n_1, \cdots, n_r) \in P_{n,r}} D_{n_1} D_{n_2} \cdots D_{n_r} .$$

Then we have

(i) $\delta_n (n = 1, 2, \cdots)$ *is a k-derivation*,

(ii) $D_n = \sum_{r=1}^{n} \frac{1}{r!} \sum_{(n_1, \cdots, n_r) \in P_{n,r}} \delta_{n_1} \delta_{n_2} \cdots \delta_{n_r}$.

Proof. (i) For $n=1$ we have $\delta_1 = D_1$ which is clearly k-derivation.