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NON-SOLVABLE GROUPS, WHOSE CHARACTER DEGREES ARE PRODUCTS OF AT MOST TWO PRIME NUMBERS*)

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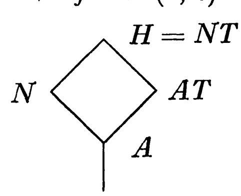
1. Introduction

If $n \in N$ has the prime-number-decomposition $n=\prod_{i=1}^{k} p_i^{a_i}$, we define $\omega(n)=\sum_{i=1}^{k} a_i$. If $\mathrm{Irr}(G)$ is furthermore the set of irreducible complex characters of the finite group G, we define $\omega(G)=\max_{\chi \in \mathrm{Irr}(G)} \omega(\chi(1))$.
Suppose first that $\omega(G)=1$, which means that all non-linear characters have prime-number-degrees. By a theorem of M. Isaacs and D. Passman (cf. Isaacs [6], 14.4), G must be solvable. But this conclusion does not hold, if $\omega(G)=2$; for example $cd(A_5)=\{1, 3, 4, 5\}$ and $cd(A_7)=\{1, 6, 10, 14, 15, 21, 35\}$ (cf. McKay [8]; cd=character degrees).
There seem to be many solvable groups G with $\omega(G)=2$. In a later paper we shall consider these; in particular we shall show that they have derived length at most 4.**)
The class of non-solvable groups G with $\omega(G)=2$ is quite small. It is completely described by the following theorem.

Theorem. *Suppose that G is non-solvable. Then $\omega(G)=2$ if and only if G is a direct product of an abelian group with a group H of the following type:*
(1) $H \cong A_7$.
(2) $H \cong A_5$.
(3) *$H=NT$, where N is a normal abelian 2-subgroup of H, $T \cong A_5$, $N=N_0 \times A$, where A is a the natural module for $SL(2, 4) \cong A_5$ and $[N, T] \leq A$.*



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**) Arch. Math. 46 (1986), 387-392.