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ON UNRAMIFIED GALOIS EXTENSIONS OF REAL QUADRATIC NUMBER FIELDS

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1. Introduction

The purpose of this note is to construct infinitely many real quadratic number fields each having an A_5-extension which is unramified at all primes including the infinite primes (abbrev. strictly unramified). Here, a G-extension means a Galois extension having G as its Galois group, and S_n (resp. A_n) denotes the symmetric group (resp. the alternating group) of degree n. In [12], Yamamoto constructed infinitely many real quadratic number fields each having an A_n-extension which is unramified at all finite primes (abbrev. weakly unramified) for each $n \geqq 4$, but they are always ramified at the two infinite primes. In this note, we shall prove the following

Theorem. *Let S_1 and S_2 be given finite sets of prime numbers satisfying $S_1 \cap S_2 = \emptyset$ and $2,5 \notin S_2$. Then there exist infinitely many real quadratic number fields F satisfying the following conditions:*
(a) *F has a strictly unramified A_5-extension.*
(b) *All primes in S_1 are unramified in F.*
(c) *All primes in S_2 are ramified in F.*

Composing such an A_5-extension with some real quadratic number field, we obtain infinitely many real quadratic number fields with a strictly unramified S_5-extension. Furthermore, we describe a method for constructing infinitely many real quadratic number fields having a strictly unramified A_n-extension for larger n, and give some examples of real quadratic number fields with class number one having a strictly or weakly unramified A_n-extension, for $n=5$, 6, and 7.

This note is based on a part of the author's Master's thesis [13].

2. Proof of the theorem

Take a polynomial of the form

$$f(x) = x^5 - 2m^2 x^3 + (6m^2 - 1)x - (m-4) .$$
$$(m\text{: a positive integer})$$