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ON BLOCKS OF FINITE GROUPS WITH RADICAL CUBE ZERO

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Let G be a finite group and k be an algebraically closed field of characteristic p, a prime number. Let B be a block algebra of the group algebra kG with defect group D and let $J(B)$ denote the Jacobson radical of B. It is well known that $J(B)=0$ if and only if $D=1$. Furthermore it is true that $J(B)^2=$ if and only if $p=2$ and $|D|=2$.

In this paper we shall prove the following theorem.

Theorem 1. *$J(B)^3=0$ (but $J(B)^2\neq 0$) if and only if one of the following conditions holds;*

(1) $p=2$, D is a four group and B is isomorphic to the matrix ring over kD or is Morita equivalent to kA_4 where A_4 is the alternating group of degree 4,

(2) p is odd, $|D|=p$, the number of simple kG-modules in B is $p-1$ or $p-1/2$ and the Brauer tree of B is a straight line segment such that the exceptional vertex is in an end point (if it exists).

For the prime 2 we have the following.

Theorem 2. *Assume $p=2$. Let U be the projective indecomposable kG-module with $U/Rad(U)=k_G$, the trivial kG-module. If Loewy length of U is 3, then a 2-Sylow subgroup of G is dihedral.*

EXAMPLE.

(1) The principal p-block of the following groups satisfies the conditions in Theorem 1.

(a) G is a four group or A_4 and $p=2$.

(b) G is the symmetric group or the alternating group of degree p and p is odd.

(2) Erdmann [6] shows that for each prime power q with $q\equiv 3$ (mod. 4) the group PSL $(2, q)$ satisfies the assumption in Theorem 2.

1. Preliminaries

In this section we shall prove some lemmas which will be used to prove