Soga, H.
Osaka J. Math.
23 (1986), 441-456

CONDITIONS AGAINST RAPID DECREASE OF OSCILLATORY INTEGRALS AND THEIR APPLICATIONS TO INVERSE SCATTERING PROBLEMS

HIDEO SOGA

(Received March 22, 1985)

Introduction

Analysing singularities of distributions, we often examine the following integral with a parameter $\sigma>0$:

$$I(\sigma) = \int_{\boldsymbol{R}^n} e^{-i\sigma\varphi(x)}\rho(x;\sigma)dx \quad (\text{or } \int e^{i\sigma\varphi(x)}\rho(x;\sigma)dx)\,,$$

where $\varphi(x)$ is a real-valued C^∞ function and $\rho(x;\sigma)$ is a C^∞ function with an asymptotic expansion

$$\rho(x;\sigma)\sim\rho_0(x)+\rho_1(x)\,(i\sigma)^{-1}+\rho_2(x)\,(i\sigma)^{-2}+\cdots \quad (\text{as } \sigma\to\infty)\,.$$

In this paper we study conditions for the integral $I(\sigma)$ not to decrease rapidly as $\sigma\to\infty$, and solve some inverse scattering problems.

As is well known, if stationary points of $\varphi(x)$ are non-degenerate (i.e. det $(\partial_x^2\varphi(x))\neq0$ when $\partial_x\varphi(x)=0$), $I(\sigma)$ is expanded asymptotically as $\sigma\to\infty$, and we can know whether $I(\sigma)$ decreases rapidly as $\sigma\to\infty$. Also when the stationary points are degenerate, the asymptotic expansion of $I(\sigma)$ is obtained if $\varphi(x)$ is analytic (cf. Varchenko [16], Duistermaat [1], etc.), and then we can know it through the expansion. But it seems difficult to do so when all derivatives of $\varphi(x)$ vanish at some points, whose case we take into consideration. In our methods we do not employ the asymptotic expansion of $I(\sigma)$. In the previous paper [13], the author examined the case that $n=2$ and $\rho_1(x)=0$ $(j\geqq1)$: If $\rho_0(x)\geqq0$ on $\boldsymbol{R}^2$ and $\rho_0(x_0)>0$ for a degenerate stationary point x_0 of $\varphi(x)$, then $(1+|\sigma|)^m I(\sigma)\notin L^2(\boldsymbol{R}^1)$ for some $m<2^{-1}$ (cf. Theorem 1 of [13]). Improving the methods in [13], whose idea is due to [8], we shall obtain similar results also in the case of $n\geqq3$.

Let supp$[\rho(\cdot\,;\sigma)]$ and supp$[\rho_j]$ $(j\geqq0)$ be contained in a compact set D in $\boldsymbol{R}^n$. We set

$$E(s) = \{x\colon \varphi(x)\leqq s\} \qquad (s\in\boldsymbol{R})\,,$$