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Osaka J. Math.
23 (1986), 363-388

CONJUGATE SETS OF GAUSSIAN RANDOM FIELDS

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(Received February 14, 1985)

1. Introduction

In [3], [4], [5], [6] and [7], P. Lévy has introduced the notion of the conjugate sets associated with Gaussian random fields (G.r.f.'s) and studied the properties of these sets. Recently, in [1] and [2], we have also shown that this notion is effective to discuss the independence structures of G.r.f.'s. In this paper, we shall be concerned with the characterization of G.r.f.'s with parameter space R^d in terms of the conjugate sets associated with them.

Let S be the class of all the functions on $[0, \infty)$ expressed in the form

$$r(t) = ct^2 + \int_0^\infty (1-e^{-t^2u})u^{-1}d\gamma(u) \qquad (t\geqslant 0)\,, \tag{1.1}$$

where c is a non-negative constant and γ denotes a measure on $(0, \infty)$ such that

$$\int_0^\infty (1+u)^{-1}d\gamma(u) < \infty \quad \text{and} \quad r(1) = 1\,.$$

An important subclass of S is given by

$$L = \{r(t) = t^\alpha;\ 0<\alpha\leqslant 2\}\,. \tag{1.2}$$

Then it is well known that for every $r(t)\in S$ and every $d\geqslant 1$ there exists a mean zero G.r.f. $X=\{X(x);\ x\in R^d\}$ with homogeneous and isotropic increments that is determined by the *structure function* $r(t)$, i.e.,

$$E[(X(x)-X(y))^2] = r(|x-y|) \qquad \text{for every} \quad x, y\in R^d$$

and

$$E[X(x)] = 0 \qquad \text{for every} \quad x\in R^d\,.$$

We can determine this G.r.f. X uniquely except for additional Gaussian random variables with mean zero. We may identify two G.r.f.'s on R^d which are determined by the same structure function, because such G.r.f.'s have the same probabilistic structure related to conditional dependence. From this point of view, we often use the notation $(X, r(t))$ instead of X. For details of these G.r.f.'s, see [2], [8], [9], [13] and Remark 2 in Section 2.