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Osaka J. Math.
23 (1986), 337-361

FINE POTENTIAL THEORY IN DIRICHLET SPACES

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(Received October 12, 1984)

Introduction. This article is an extended and revised version of a part of my thesis [17, §§ 6–8], and it should be seen as part two of [19]. It continues the study of potential theory with an emphasis on fine-topological questions from [18, 19], in the framework of (symmetric) Dirichlet spaces. In [19], we treated some questions on capacitary integrals and related matters. Here we present a rather general theory for superharmonic functions in Dirichlet spaces. (Some results were proved in [18].) As background serves a problem on bounded point evaluations for BLD-functions, harmonic on a certain set, see [14], and Fuglede's work on finely superharmonic functions [9], in particular their relations to certain functions in the space BLD, the archetype for all Dirichlet spaces, treated in [10]. (An application of the theory developed here is given in [21].)

Dirichlet spaces were originally introduced in the late 'fifties by Beurling and Deny. At about the same time, Hunt prepared the way for a general probabilistic potential theory. In the translation invariant case, i.e. in the case of Markov processes with stationary, independent increments, it is not hard to establish a one-to-one correspondence between (sufficiently smooth) symmetric Markov processes and (sufficiently smooth) Dirichlet spaces with translation invariant norm. Fukushima realised that if one looked upon the semi-groups involved as operators on L^2 —and not, as had been customary, on some class of continuous functions— a similar result was valid in general: under some mild smoothness assumptions there is a right-continuous strong Markov process (in fact, a Hunt process) to every Dirichlet space, and vice versa. (Here the smoothness assumptions are put directly on the Dirichlet space, while usually in potential theory one assumes that the semi-group (or the Green operator) is smooth in that it produces smooth functions.) We will use this correspondence whenever convenient and refer to Fukushima's book [11] for details.

The Dirichlet space approach to harmonic functions is by orthogonal projections, Dirichlet's principle from a modern point of view. The difference

(1) Supported in part by the Swedish Natural Science Research Council, NFR.